Onboard Pseudolite Augmentation System for Relative Navigation *

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BIOGRAPHY

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1 ABSTRACT

GPS carrier phase positioning can provide accurate measurements of relative position and attitude between multiple vehicles. In environments without adequate signal coverage from the NAVSTAR GPS satellites, the available signals can be augmented with pseudolites onboard the vehicles in the formation. This paper experimentally demonstrates the feasibility of such a system, with a formation of three vehicles. In addition, this paper examines some of the robustness issues with such a setup, specifically in dealing with re-initialization of the carrier phase integers.

2 INTRODUCTION

Precise relative navigation is a necessary technology for any operation involving multiple vehicles working together in a coordinated manner. One practical example of such coordinated behavior is formation flying of spacecraft. Many missions have been proposed [1, 2, 3] which consist of several spacecraft working together to achieve a common objective such as imaging a star. For this to be successful, a precise sensor of relative positions and attitudes between all vehicles is required.

Carrier phase differential GPS positioning has been shown to be a very accurate technique for estimating relative position and attitude between multiple vehicles [4, 5, 6]. Yet many locations in which formation flying missions would be advantageous are not well covered by the NAVSTAR satellites. For example, in high orbits such as GEO, satellite reception is often not sufficient to resolve position and attitude between all vehicles in the formation. One method of taking advantage of the GPS system in areas with inadequate signal coverage is to augment the available signals with additional signals from pseudolites. Several different techniques have been suggested for creating such an augmentation system, including using pseudolites in known locations on the ground that broadcast to space [7], and placing a network of pseudolites in a MEO or GEO orbit [8]. This paper presents an alternative approach of mounting a GPS transceiver system onboard each of the vehicles in the formation, such that each vehicle can transmit and receive GPS signals, and communicate GPS data with one another. While similar ranging systems have been proposed for operation in the Ka band [1], this research concentrates on currently available hardware, and incorporates NAVSTAR satellites that are visible to the formation. The augmentation system is independent of any external infrastructure, and can therefore can be used anywhere. Formation flying using the transceiver system was originally proposed in [9], and is further developed in this paper.

Previous experimental work is extended to a formation of three vehicles, each equipped with a transmitter, receiver, and communication link. Dynamic positioning results for the formation are presented. In addition, the question of robustness is examined in detail. As satellites drop in and out of view, the carrier phase integers must be re-initialized. The satellites will come in and out of view not only from changes in line-of-sight to the satellites, but also as a result of poor signal tracking in a high multipath, noisy environment. Note that locating the transmitters onboard the vehicles contributes to the noise. Thus, re-initialization of carrier phase integers must be an integral part of the positioning problem, and must be handled in a robust manner to ensure longevity of the mission.

Two re-initialization techniques are examined. One is implemented on the experimental system, and re-

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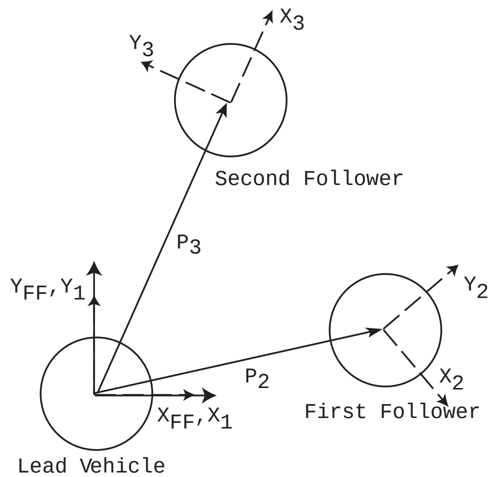

Fig. 1: Coordinate System for Formation

solves carrier phase biases to a floating point value with a null space algorithm. The second method is a layered integer resolution technique which combines a quickly available floating point estimate of the integer bias with a less frequently available integer search solution for additional accuracy. The layered approach benefits from the accuracy improvement provided by fixing biases to an integer value. Depending on the rate at which re-initialization must occur, one method will be preferable over the other. For slower re-initialization rates, the floating point bias filter is accurate and less complicated than the layered approach. For more rapid re-initialization rates, the layered approach prevents slow growth in bias estimate errors. Several metrics are used to divide regimes of slow and fast re-initialization rates. Empirical values are given for these metrics for the formation flying testbed, but these values will vary for other experimental systems. For example, with increased vehicle motion, observability of the integers is improved, which will impact the distinction between slow and fast re-initialization. This research focuses on cases of limited relative vehicle motion, which is representative of fuel-critical space missions.

3 PROBLEM DESCRIPTION

The proposed pseudolite augmentation system is capable of providing only relative positioning information. Therefore, it is assumed that one vehicle, the lead vehicle, will have knowledge of its absolute position or orbital elements. In addition, it is assumed this vehicle will be outfitted with an absolute attitude sensor. These assumptions are only *necessary* if the NAVSTAR constellation satellites are to be included in the solution, but in general, it is desirable to have knowledge of the absolute location of the formation. Position and attitude of all other vehicles in the formation will be provided by the pseudolite augmentation system.

A formation of three vehicles is considered. The coordinate system employed is fixed with the lead vehicle, such that the local formation frame (FF) coincides with the lead vehicle frame (X_1, Y_1) (see Figure 1). The other two follower vehicles are defined in the local formation frame. The state variables of interest are therefore the position vectors to the center of mass of each of the follower vehicles (P_2, P_3), as well as the elements of the rotation matrices defining the rotations from the formation frame to the follower vehicle frames. For a three dimensional positioning solution, this description yields three position unknowns for each follower vehicle, and three rotation parameters for each follower vehicle, for a total of $6N$ unknown parameters, where N is the number of follower vehicles (in this case three). For the results presented, the solution is carried out in two dimensions. The ability of the sensor to provide full three dimensional position solutions has been shown in [4, 9], thus for this work, only the parameters important in the two dimensional testbed have been included in the unknown state. The rotations of the vehicles are described using quaternions, limited by the knowledge that the rotations are purely in yaw. The resulting unknowns are

$$X_i = \begin{bmatrix} x_i & y_i & e_{3_i} & e_{4_i} \end{bmatrix} \tag{1}$$

for each follower vehicle i, where x and y comprise the range vector P_i from the lead vehicle to the ith follower vehicle, and e_{3_i}, and e_{4_i} comprise the quaternions describing the yaw rotation from the formation frame to the ith vehicle frame (e_{0_i} and e_{1_i} are assumed to be zero, since the rotation is only in yaw).

There are two types of available measurements: single differences and double differences. This research was conducted with a TANS Quadrex receiver, therefore the available measurements are based on the assumption of a receiver with multiple RF inputs to allow for four antennas, or three baselines, per vehicle. Single differences are formed with two antennas on the same vehicle and one transmitter (*transmitter* refers to both GPS satellites and onboard pseudolites). Double differences are taken between two transmitters and two antennas each on a separate vehicle. The single differences yield information primarily about the relative attitude of the vehicles, while the double differences yield information primarily about the relative range. However, depending on the range between vehicles, coupling exists between attitude and positioning measurements. For the ranges used in this experimental work (from 2 to 10 feet separation between vehicles), the coupling is strong. For the single differences, all clock biases between the transmitter and receiver cancel out. For the double differences, the samples on two separate receivers must be taken at exactly the same time. This is accomplished by synchronizing the receivers within 1 millisecond by a data message broadcast by a master pseudolite [6]. Further synchronization is achieved through interpolation. The master pseudolite is located on one of the three vehicles, and all double differences are formed with the master pseudolite as one of the two transmitters. Using a master pseudolite instead of GPS time for synchronization obviates the need for special handling when all NAVSTAR satellites drop out of view. The set of equations forming the measurement vector $h(X)$ are written in Equation 2.

$$\begin{bmatrix} \Delta\phi_{2j}^{k} \\ \vdots \\ \Delta\phi_{3j}^{k} \\ \vdots \\ \nabla\Delta\phi_{12}^{mk} \\ \vdots \\ \nabla\Delta\phi_{23}^{mk} \\ \vdots \end{bmatrix} = \begin{bmatrix} h_{sd}(X_2, X_3) \\ \vdots \\ h_{sd}(X_2, X_3) \\ \vdots \\ h_{dd}(X_2, X_3) \\ \vdots \\ h_{dd}(X_2, X_3) \\ \vdots \end{bmatrix} + \lambda N + \begin{bmatrix} \beta_{2j} \\ \vdots \\ \beta_{3j} \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \end{bmatrix} + \nu \quad (2)$$

which is in the form of

$$y = h(X) + \lambda N + \bar{\beta} + \nu \quad (3)$$

N is a vector containing the integer biases, $\bar{\beta}$ is the vector of line biases for each vehicle and baseline (β_{ij}) accompanying the single differences, and ν represents the noise in the measurements. For the single differences, the superscripts on $\Delta\phi$ represent transmitters, and the subscripts represent, in order, vehicle and antenna, *i.e.* $\Delta\phi_{2j}^{k}$ represents the single difference measurement from transmitter k to the jth antenna of vehicle 2. The second antenna in the single differences is always the master antenna, given the TANS Quadrex master-slave configuration. Single differences with the lead vehicle are not included in the measurement vector, since the attitude of the lead vehicle is not being estimated. For the double differences, the superscripts on $\nabla\Delta\phi$ represent the two transmitters involved in the difference, with the m representing the master pseudolite, and the subscripts represent the two receiving vehicles. $h_{sd}(X_2, X_3)$ is a nonlinear equation geometrically representing the single difference measurement, and is a function of X_2 and X_3, as defined in Equation 1. $h_{dd}(X_2, X_3)$ is a nonlinear equation representing the geometry of the double difference equation. As an example, $\Delta\phi_{2j}^{3}$ is written out in entirety below.

$$\begin{aligned} \Delta\phi_{2j}^{3} = & \left[\left| (P_2 + R_{FF}^{2} B_{2m}) - (P_3 + R_{FF}^{3} Q_3) \right| \right] \\ & - \left[\left| (P_2 + R_{FF}^{2} B_{2j}) - (P_3 + R_{FF}^{3} Q_3 \right| \right] \end{aligned}$$

where P_i is the range vector to the ith vehicle, R_{FF}^{i} is the rotation matrix from vehicle i's frame to the formation frame, B_{ij} is the baseline to the jth antenna on the ith vehicle, and Q_i is the baseline to the transmitter on the ith vehicle. This equation is valid for an onboard pseudolite transmitting to two antennas on a vehicle. For a satellite, the equation will be modified slightly.

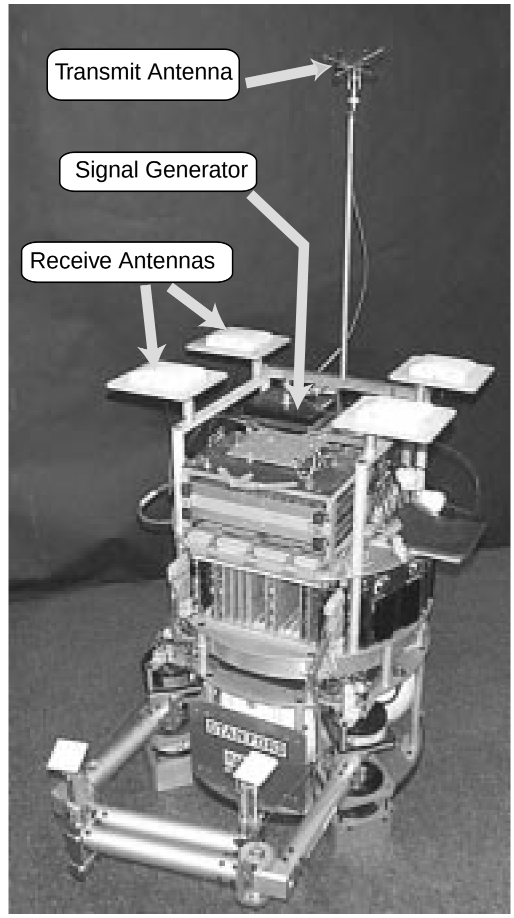

Fig. 2: One of Three Vehicles in the Formation Flying Testbed, Equipped with GPS Transmitter and Receiver

4 EXPERIMENTAL TESTBED

To study formation flying issues, a formation flying testbed has been created in the Stanford Aerospace Robotics Laboratory. It consists of three active free-flying vehicles that move on a 12 ft × 9 ft granite table top (see Figure 2). These air cushion vehicles simulate the zero-g dynamics of a spacecraft formation in a plane. The vehicles are propelled by compressed air thrusters, and momentum wheels. Each vehicle has a PowerPC processor.

An overhead vision system is mounted above the table to provide a truth sensor for the system. Each object on the table is tagged with a unique LED pattern that can be tracked by the vision system. This system has an absolute accuracy better than 1 cm for position and 0.5 deg for attitude throughout the workspace.

An indoor GPS environment was originally con-

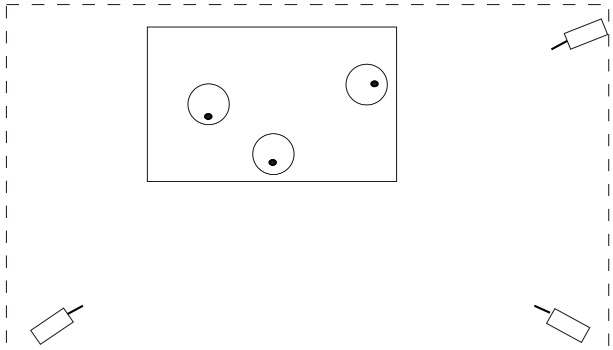

Fig. 3: Layout of Transmitters in GPS Testbed. Three Ceiling Mounted Pseudolites Around Perimeter, Three Robots (circles) with Transmitters Onboard.

structed for the experiments by Zimmerman [6]. This consisted of ceiling mounted pseudolite transmitters broadcasting the L_1 carrier phase signal modulated by a unique C/A code, with no navigation data. Three of these pseudolites are still in use, representing the available NAVSTAR satellites. Since the testbed is located in the basement of a building, the actual satellite signals cannot be received. The remaining GPS signals are provided by the transceiver systems onboard each vehicle. The geometry of the problem is shown in Figure 3.

The transceiver system consists of a six channel TANS Quadrex receiver, a GPS signal generator, and a wireless ethernet for communication. All GPS signals are produced using Integrinautics signal generators (see Figure 4). The pseudolites operate in a pulsed mode, with a one millisecond period, and a duty cycle of approximately 14%. No direct synchronization is utilized between different pseudolites. Instead, each pseudolite pulses in a pseudo-random manner to avoid interference with other pulsing pseudolites. This scheme works well for the formation size considered in this work, however for a significantly larger formation, some form of synchronization is desirable. The purpose of the pulsing is to prevent jamming the receivers. Since the transmitters are located in close proximity to the receivers (within 10 cm), if the onboard pseudolites were to transmit continuously, the receivers would jam.

Three different types of antennas are used in this work. The ceiling mounted pseudolites transmit through narrow beamwidth helical antennas, that have been directed at the workspace. The receive antennas on each vehicle are standard GPS patch antennas. For onboard transmission, a Lindenblad antenna is employed (Figure 5). The Lindenblad antenna provides an omnidirectional right-hand circularly polarized signal, which can be received by the other vehicles, independent of the orientation of the transmitting vehicle.

Fig. 4: Integrinautics IN200c Signal Generator

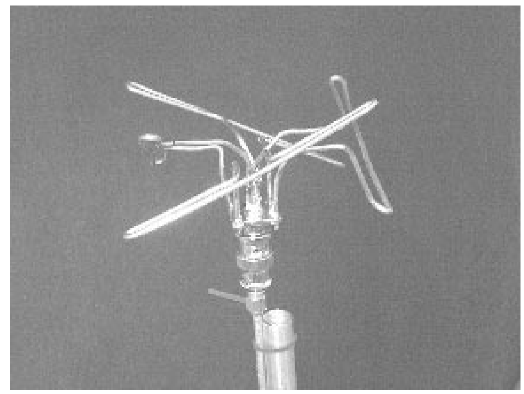

Fig. 5: Lindenblad Antenna.

The antenna placement of both receive and transmit antennas has been optimized for the current testbed, and can be viewed in Figure 2. For longer baselines between vehicles, the receive antennas would need to be tilted towards the other vehicles, and placed around the sides of the vehicles [10].

5 DYNAMIC TESTING

The results presented in this section are from tests carried out with three vehicles, three onboard transmitters, and three ceiling mounted pseudolites. At any given time, all six channels are not guaranteed to be in lock, in fact as few as three transmitters may be in lock. The data has been collected during dynamic tests, in which the vehicles are undergoing motions not exceeding 30% of the baseline separation between vehicles. In this harsh indoor environment, small motions result in large changes in the signal environment, thus small motions are adequate for testing the system performance under rapidly varying signal levels.

At startup, initialization of the carrier phase integer ambiguities is achieved with the vision system. Previously (see [9]), it has been shown how the formation integers can be initialized with motion of the vehicles. In addition, methods of resolving the initial set of integers for a formation of vehicles have been discussed in [11] and [12]. Thus in this paper, emphasis has been placed not on initialization, but on re-initialization. As transmitters fade in and out of lock or are obscured, the carrier phase biases must be re-initialized. This is handled by a continuously running re-initialization filter based on a null space algorithm. The filter separates bias estimation from position estimation, and produces a floating point estimate of the unknown biases. The algorithm is described in detail in Section 6.

Typical motions incurred during data collection are plotted in Figures 6 and 7, which show respectively the position in time of the first follower vehicle in x and y coordinates of the formation frame (see Figure 1). The vision relative position estimate and the GPS position estimate are superimposed, to demonstrate the accuracy of the GPS solution throughout the maneuver. Change in yaw for the first follower vehicle is depicted in Figure 8. Again, both the vision and GPS attitude solutions are presented. The mean error values and standard deviations for relative position and yaw are listed in Table 1, in the column labeled Data 1. Relative positioning accuracies for a second data set are shown in Figures 9 and 10. This data was collected over a two minute period of time. Error in the x axis estimate is plotted against error in the y axis estimate for both follower vehicles. The means and standard deviations of the errors are listed in Table 1, in the column labeled Data 2.

These data sets illustrate the geometric dependence of the solution accuracy, as well as a shortcoming in the current bias re-initialization algorithm. The accuracy obtained from a position solution will vary depending on the location of the vehicles relative to one another. Baseline geometries should be chosen to ensure good geometric dilution of precision (GDOP). For the geometry used in these tests, the DOP for relative position to the second vehicle was on average 10% worse than for the first follower vehicle. However, the more significant factor in the positioning errors is insufficient integer resolution. Too rapid a change in satellite lock causes degradation in positioning estimation. With frequent changes in available measurements, the carrier phase biases do not fully converge before re-initialization occurs again.

The result of inadequately resolved biases can be seen in these sets of data. Looking at scatter in the relative position errors for the second data set (Figures 9 and 10), a clear pattern can be seen in the errors to both vehicles: the data is divided into clusters. This is shown more explicitly in Figures 11 and 12, which are the histograms of the error in x and y position to the first follower vehicle in the formation frame. The distributions are bimodal, instead of the expected unimodal distribution. When satellites come rapidly in and out of lock, the correct biases are not sufficiently resolved before a new bias must be found. Thus, the algorithm does not have enough time to converge, and

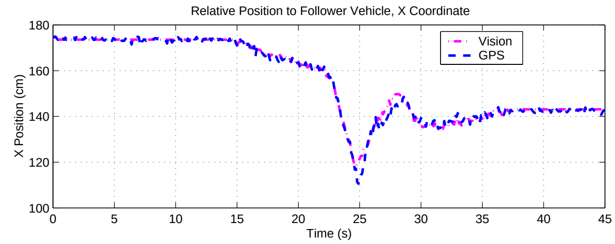

Fig. 6: GPS and Truth Sensor Measurements of X Coordinate to First Follower, Re-Initialization with Null Space Algorithm, Data 1

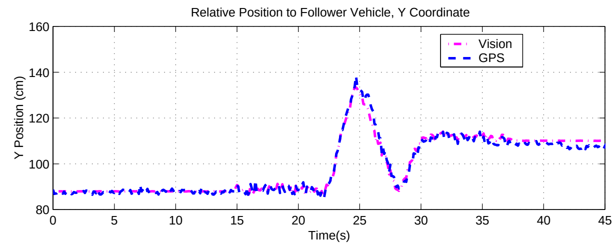

Fig. 7: GPS and Truth Sensor Measurements of Y Coordinate to First Follower, Re-Initialization with Null Space Algorithm, Data 1

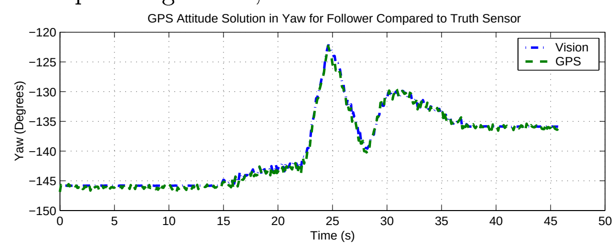

Fig. 8: GPS and Truth Sensor Measurements of Yaw of First Follower, Re-Initialization with Null Space Algorithm, Data 1

very little data is available for correctly resolving the bias. The resulting growing error in the bias estimates for the double difference measurements is shown in Figure 13 and the error in the single difference biases is shown in Figure 14. The errors in double difference bias estimates are larger than those for single difference bias estimates since the double difference biases are less observable for small motions. The jump in the bias error occurring at approximately 70 seconds results in the split distribution of the positioning errors for the second data set.

This suggests the need for a better way to handle integer re-initialization under circumstances of short time and small motion between changes in available measurements. Three different metrics are listed in Table 1 for three data sets, in order to define a regime of rapid re-initialization rate, in which methods other than the current algorithm would improve positioning accuracy. The first metric for rate of re-initialization in Table 1, ρ_{avg}, is the average percentage of the double difference measurements with unresolved biases for any given epoch. N_{avg} is the average total number of integer resets that occur per minute for double differ-

Table 1: Comparison of positioning errors for three different data sets with differing re-initialization frequencies

	Data 1		Data 2		Data 3	
Variable	Mean	Std	Mean	Std	Mean	Std
X_2 (cm)	0.36	1.62	0.81	1.26	0.34	0.49
Y_2 (cm)	0.39	1.53	0.49	1.13	0.07	0.54
θ_2 (deg)	0.25	0.35	0.3	.65	0.017	0.32
X_3 (cm)	2.32	2.39	1.66	1.55	0.51	0.63
Y_3 (cm)	0.87	1.29	2.5	1.28	0.43	0.67
θ_3 (deg)	0.52	0.56	1.18	0.84	0.43	0.26
ρ_{avg} (% unresolved/epoch)	2.5	6.7	8.7	14.9	0.4	3.0
N_{avg} (reset/min)	17.5		26		5	
ν_{meas} (reset/min)	3.34		3.89		0.85	

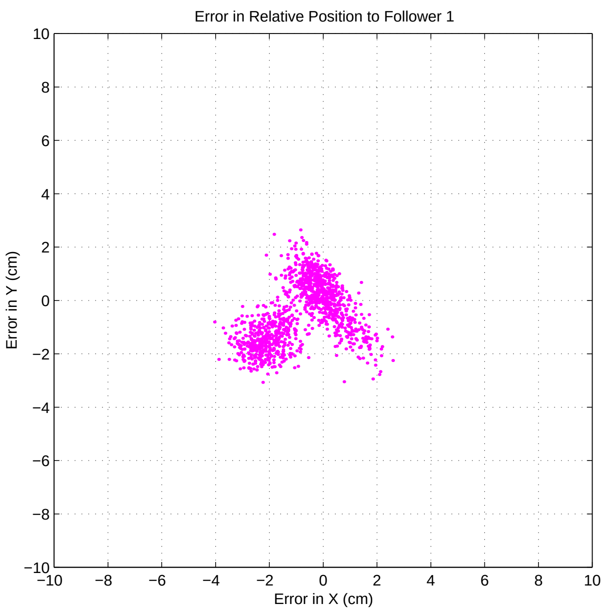

Fig. 9: Error in Relative Position Estimate to First Follower Vehicle,Frequent Re-Initialization with Null Space Algorithm, Data 2

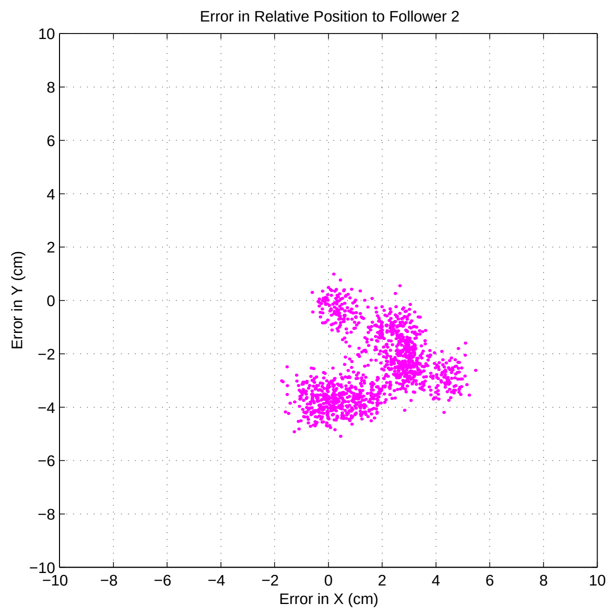

Fig. 10: Error in Relative Position Estimate to Second Follower Vehicle, Frequent Re-Initialization with Null Space Algorithm, Data 2

ence measurements. Finally, ν_{meas} is the average number of resets per minute for individual double difference measurements. Emphasis has been placed on the double difference measurements since these are critical for relative range estimation, and as shown in Figure 13, the double differences are slow to converge due to small vehicle motions. For comparison with the first two data sets, the third column of data in Table 1 lists statistics for a set of data with fewer re-initializations, for which the null space algorithm performs well. Figure 16 shows the positioning accuracies to the first follower vehicle for this set of data. Comparing the re-initialization metrics between all three data sets, N_{avg}, the mean of ρ_{avg}, and ν_{meas} for data set 3 are all at most 30% of the values for the first two data sets. Several other data sets with slow re-initialization rates and good performance of the null space algorithm were also examined. Based on these observations, empirical values for dividing rapid re-initialization from slow re-initialization were developed. Of the data sets with good performance of the null-space algorithm, data set 3 had the highest values for the metrics. Therefore heuristics for defining rapid re-initialization were based on data set 3: *rapid re-initialization* refers to $N_{avg} \geq 5, \nu_{meas} \geq 0.85$, and $\rho_{avg} \geq 0.4\%$.

6 RE-INTIALIZATION

Continual re-initialization of the carrier phase integers is a problem inherent to high multipath environments, extreme near-far environments, and other situations in which satellites will be quickly coming in and out of view. The carrier phase integers of the newly acquired measurements must be quickly resolved. With a sufficiently overdetermined estimation problem, the measurements with resolved integers may be used to form $\hat{x}$, an estimate of the position state, which in turn can be used in forming $\hat{\phi}$, a floating point estimate of the unbiased measurements. The integer vector for all measurements can then be easily formed as $\hat{N} = \phi - \hat{\phi}$. For cases of frequent re-initialization, error is inherently introduced into the

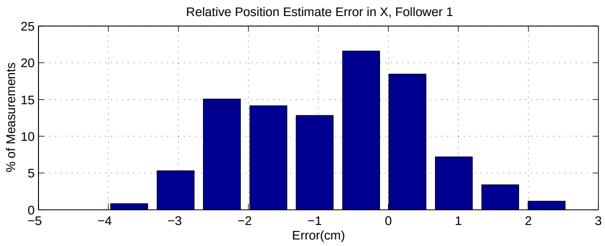

Fig. 11: Error Distribution in X Coordinate of Position of First Follower, Data 2

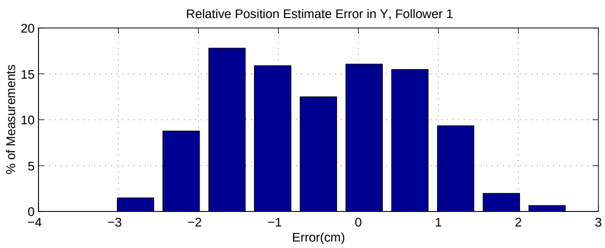

Fig. 12: Error Distribution in Y Coordinate of Position of First Follower, Data 2

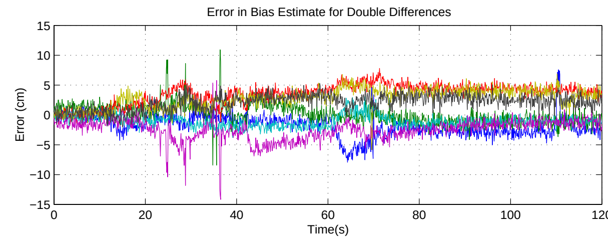

Fig. 13: Error in Bias Estimate for Double Difference Measurements, Frequent Re-Initialization with Null Space Algorithm, Data 2

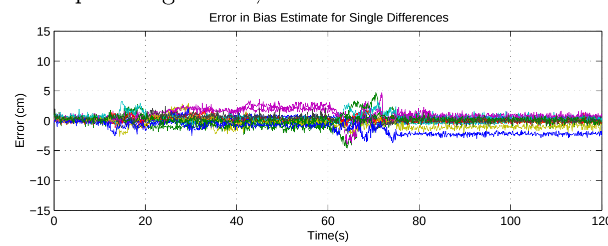

Fig. 14: Error in Bias Estimate for Single Difference Measurements, Frequent Re-Initialization with Null Space Algorithm, Data 2

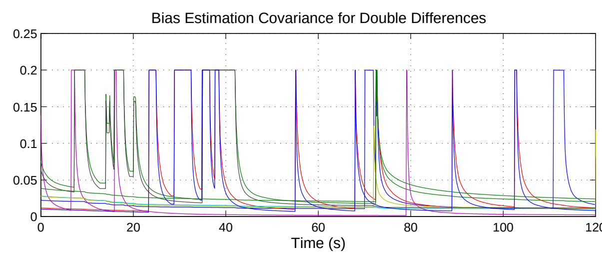

Fig. 15: Error Covariance for Double Difference Bias Estimates, Null Space Algorithm, Data 2

system. An alternative approach is one based on an algorithm used in the Integrity Beacon Landing System (IBLS) work [13]. The performance of a modified version of this algorithm was shown in Section 5. With rapid re-initialization (as defined in the previous section, $N_{avg} \geq 5, \nu_{meas} \geq 0.85$, and $\rho_{avg} \geq 0.4\%$) and with little motion in the system, the error in the biases slowly increases, resulting in degraded position state estimates. However, if loss of lock is less frequent, the algorithm performs well as was shown in Figure 16. This section introduces the algorithm in greater detail, then examines a more robust method of handling small motion, rapid re-initialization situations.

6.1 CURRENT ALGORITHM

The basic measurement equation (Equation 2) can be linearized about a nominal guess of the state and biases, $x_\circ, N_\circ$, and $\beta_\circ$.

$$y = h(x_\circ) + H(x_\circ)\delta x + \lambda N_\circ + \lambda \delta N + \beta_\circ + \delta\beta + \nu \quad (4)$$

or,

$$z = H(x_\circ)\delta x + \lambda \delta N + \delta\beta + \nu \quad (5)$$

in which $z = y - h(x_\circ) - \lambda N_\circ - \beta_\circ$, and H is the Jacobian of the measurement vector $h(X)$. By premultiplying both sides of the equation by L, the left null space of H, the resulting equation is

$$Lz = L\lambda\delta N + L\delta\beta + L\nu. \quad (6)$$

Grouping together the line bias terms with the integer valued biases, the equation reduces to

$$Lz = L\tilde{N} + L\nu \quad (7)$$

where $\tilde{N}$ is a combination of δN and $\delta\beta$. Equation 7 is in the form of a standard measurement equation with $\tilde{N}$ as the state to be estimated. The basic concept is to iterate between position estimation and bias estimation at each time step using the above equation to solve for $\tilde{N}$. The error statistics in the two estimation problems are linked, with $R_{pos} = \sigma^2 I + P_N$, where P_N is the error covariance of the integer estimate, R_{pos} is the measurement noise covariance for the position measurements, and σ^2 is the variance of the carrier phase measurement noise. For greater detail on this algorithm, see [13].

A case of rapid re-initialization was simulated using the null space algorithm, with re-initialization occurring at every six timesteps (mean $\rho_{avg} = 2.2\%$, $N_{avg} = 20$, and $\nu_{meas} = 1.6$ if each timestep is one second). The resulting accuracy of position to the second follower vehicle is plotted in Figure 17. The accuracies of the simulation are similar to the experimental data, during data runs with high frequency of re-initialization. The corresponding error in the bias estimates is plotted in Figure 18. The bias convergence rate is much slower than the frequency of changes in

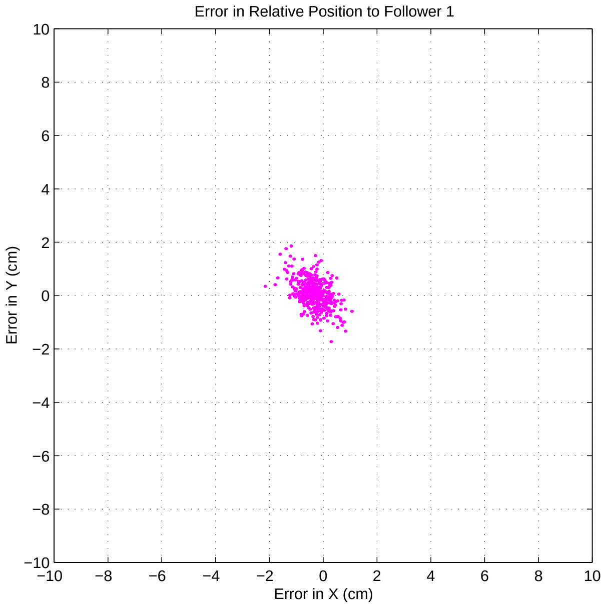

Fig. 16: Positioning Errors With Infrequent Re-Initialization Using Only Null Space Algorithm for Re-Initialization, Data 3

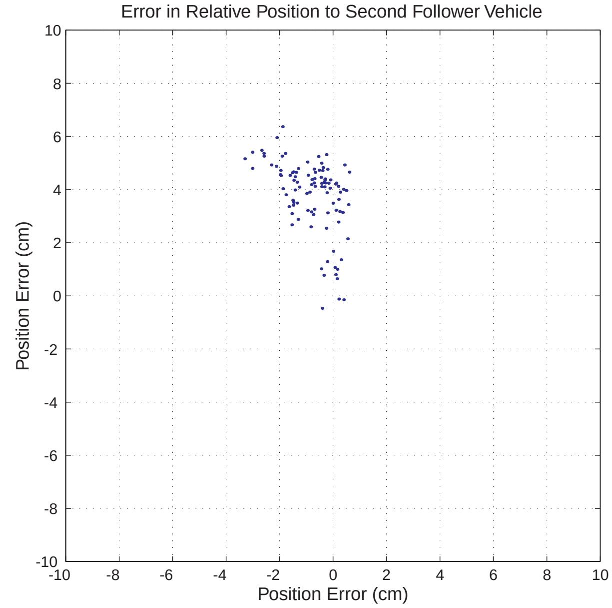

Fig. 17: Simulated Error in Position To Second Follower Vehicle, Frequent Re-Initialization with Only Null Space Algorithm

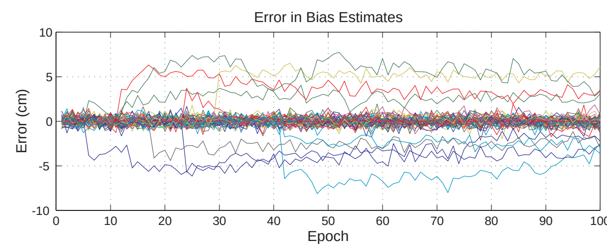

Fig. 18: Simulated Error in Bias Estimate for Frequent Re-Initialization with Only Null Space Algorithm

measurements, with the double differences as the slowest converging measurements.

6.2 PROPOSED ALGORITHM

In cases where the biases truly are integers, position estimates employing fixed bias values (biases that have been resolved from a floating point estimate to the integer value) are more accurate than those based on floating point bias values. A significant volume of work has been generated on methods for searching for the true integer value (e.g. see [14, 15, 16]). A common thread between all of these approaches is that they are both computationally and time intensive. For dynamic environments, it is desirable to have a bias estimate available at the same rate as information is provided by the GPS receiver. However, by employing a layered integer re-initialization algorithm, the advantages of a fast floating point estimation algorithm together with a very accurate integer search algorithm can be realized.

The proposed algorithm consists of three layers. Running on the bottom layer at a lower priority than the other processor tasks, is an integer search algorithm. The middle layer consists of the null space algorithm that is responsible for producing a floating point estimate of the biases. The top layer produces a rough estimate of the unknown bias based on the current best guess of the position. The top layer is more conceptual than the other layers: in reality, it may run at the same rate as the null space algorithm. However, if the position estimation is to be run at rates faster then 10 Hz, then it may be desirable to fully separate the top and middle layers, running the null space algorithm at a slower rate.

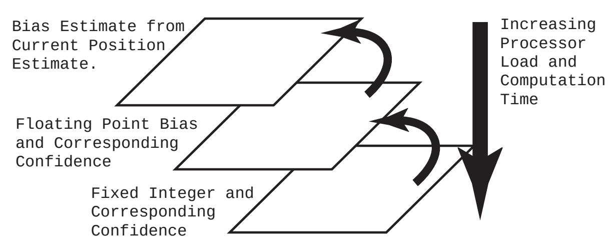

Fig. 19: Layered Integer Resolution Technique

A diagram of the layered algorithm is shown in Figure 19. As information is available from the integer search algorithm and the null space algorithm, it is passed to the position estimator, and incorporated into the solution.

The same simulated data as shown in Section 6.1, was processed with the layered integer search algorithm. The resulting accuracy of the position solution is shown in Figure 20, and the error in the measurement biases is shown in Figure 21. By fixing the biases to their integer values, greater accuracy is achieved, and divergence of the solution is prevented, even in times of rapid re-initialization.

For the layered approach to be useful, the rate of loss of *individual* integers (*i.e.* ν_{meas} in Table 1) should be less than the rate of integer convergence. If, on average, individual measurements must be re-initialized at a faster rate than the search algorithm can find the correct integer, this method will not improve the accuracy of the position solution.

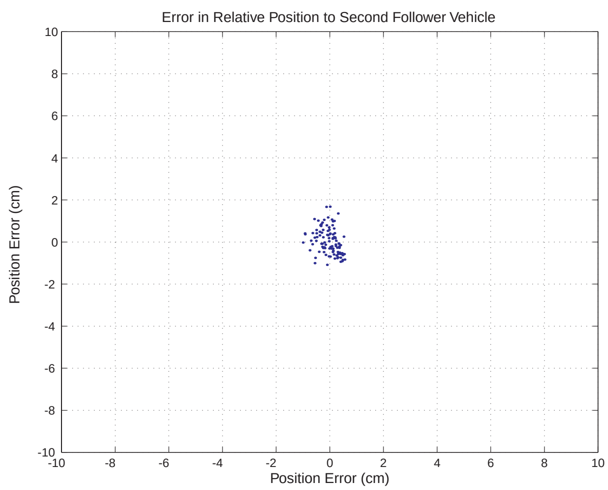

Fig. 20: Simulated Error in Position to Second Follower Vehicle During Frequent Re-Initialization, Biases Fixed with Layered Re-Initialization Algorithm

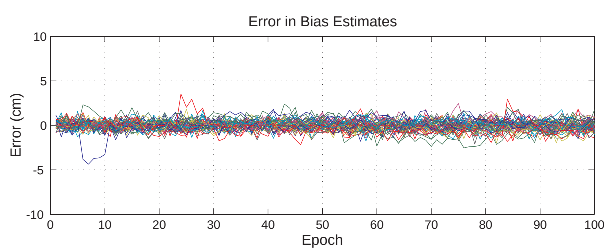

Fig. 21: Simulated Error in Bias Estimate During Frequent Re-Initialization, Biases Fixed with Layered Re-Initialization Algorithm

7 IMPLEMENTATION ISSUES

In order to implement the integer search algorithm, the unknown carrier phase biases must be integer in nature. While the double differences are integer valued, the single differences contain a non-integer valued line bias. This can typically be solved for off-line and assumed to remain constant, yet there are many applications in which the line bias does not remain constant. The onboard transmitter lends itself to a unique method for solving line biases in real time. By design, the transmitted signal can be received by the same vehicle that is transmitting. Since the distance from transmitter to receive antenna is fixed and known, the received signal can be compared against the known distance to calibrate the line biases for all baselines. Figure 22 shows the constant value of the measurement from the onboard transmitter, while the vehicle is undergoing significant motion. The signal is therefore steady enough to be used in a line bias solution.

For the testbed used in this research, the values of the line biases do not change significantly during operation of the vehicles. Thus, at startup, the received measurements are used to produce an averaged correction to the current estimate of the line biases, over a period of several minutes. This correction is incorporated into all future single difference measurements. For applications with large temperature variations, the filter could be run continuously.

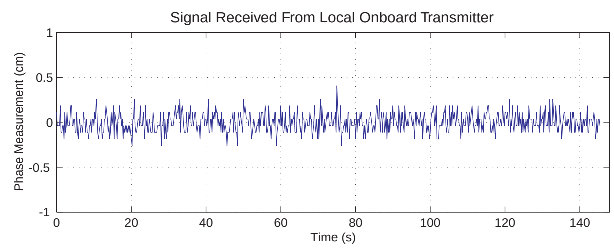

Fig. 22: Single Difference Measurement Between Transmitter and Receiver on Same Vehicle

8 CONCLUSION

Onboard pseudolites provide a convenient method of augmenting the available signals from the NAVSTAR constellation within a local formation of vehicles. With the additional signals, the relative position and attitude between all vehicles can be resolved. Positioning accuracies will vary depending on the geometry of the problem. The experimental results presented here for a formation of three vehicles show positioning accuracies within several centimeters, and a maximum attitude error of 1.2 degrees (average error of half a degree), even in extremely noisy environments.

This performance level is achievable with a very simple transceiver architecture. No formal connection exists between the receiver and transmitter, simplifying the necessary hardware. A more complicated approach does not provide any advantage, given the ability to receive the signal that is transmitted on the same vehicle. In fact, this approach provides a novel method for line bias resolution in real time.

The results bring rise to the need for a robust reinitialization algorithm. Any application in an environment with constant reacquisition of signals must quickly and accurately re-initialize the integer biases without allowing significant error to build in the system. A null space algorithm that resolves the bias to a floating point value is currently implemented on the experimental system and is promising for cases of slow re-initialization. With more frequent re-initialization, a new layered approach of integer resolution is suggested, in which an integer search algorithm is added. Several critical parameters have therefore been identified for robustness of the augmentation system: percentage of measurements with unresolved biases, total number of resets that occur in a given time period, and the average amount of time in between re-initialization for a single measurement. Ongoing work is aimed at quantifying these parameters for the environments in which formation flying missions are targeted.

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