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Robust $\mathcal{H}_\infty$ Controller Design for Systems with Hysteresis Nonlinearities

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Abstract

This paper presents a new design technique for robust control of systems having hysteresis nonlinearities using an LMI framework. Included also is an extension of recently developed stability criteria to remove the constant eigenspace associated with the multiplier used in the analysis procedure. This extension allows for the direct use of commonly available interior point methods to evaluate system stability and performance and subsequently to solve for the robust controllers. The resulting controllers are of reduced order since the additional states associated with the analysis have been eliminated. In this way there is no penalty for the stability guarantee. The effectiveness of the $\mathcal{H}_\infty$ design procedure is illustrated with a simple loop shaping control problem for a non-minimum phase system.

1 Introduction

Hysteresis is a property of a wide range of physical systems and devices, such as electro-magnetic fields, mechanical ball bearings, and electronic relay circuits. Stability analysis for systems with hysteresis is complicated by the fact that the hysteresis input-output relation is multivalued and has memory. In early work, Barabanov [1] introduced a stability multiplier of the form $\frac{\alpha+\beta s}{s}$ that can be used in the frequency domain to test for the stability of linear systems having these nonlinearities. Recent work [2] suggested this analysis be approached in an Integral Quadratic Constraint (IQC) setting, while the authors in [3] develop robust stability tests in a passivity [4] framework and express them in the form of linear matrix inequalities. In this paper we extend the analysis technique given in [3] to the problem of synthesis of robust controllers for systems containing hysteresis nonlinearities. Using recently developed approaches [5] for control design, we develop an iterative algorithm to synthesize controllers in a linear matrix inequality (LMI) framework. The controllers will guarantee simultaneous stability and performance for systems in this class of nonlinearities. In addition, we specialize the analysis [3] to a particular class of linear systems that obey a certain residue condition. For these systems we show that the constant eigenspace introduced by the multiplier can be removed from the analysis. The benefit is twofold. First, this allows direct use of currently available LMI software packages [6] which do not allow systems having any zero eigenvalues. Use of this software would otherwise require the constant eigenspace be approximated by a stable space characterized by a small, stable eigenvalue and, as a consequence, the subsequent LMI solutions would only be approximate. In contrast, the removal procedure we introduce leads to exact LMI solutions. The second benefit is that the controllers designed for these systems are of the order of the original plant and do not require the additional states commonly associated with the stability multiplier. In essence, there is no penalty for the robustness guarantee. In this paper we will use the standard 3−port, robust control design framework, as depicted in Figure 1. We will restrict attention to hysteresis, $\Phi : \mathcal{L}_{2e} \rightarrow \mathcal{L}_{2e}$, with input-output mappings having counter-clockwise circulation. Hysteresis with this characteristic is sometimes called *passive*; examples include the hysteretic relay, backlash, and Preisach-type nonlinearities [7]. The plant $G(s)$ is taken to be linear time invariant (LTI), given by the state space realization

$$G(s) \stackrel{s}{=} \left[\begin{array}{c|c|c|c} A & B_p & B_w & B_u \\ \hline C_q & D_{qp} & D_{qw} & D_{qu} \\ \hline C_z & D_{zp} & D_{zw} & D_{zu} \\ \hline C_y & D_{yp} & D_{yw} & D_{yu} \end{array}\right], \tag{1}$$

and the linear controllers $K(s)$ considered are strictly proper, defined by the state space system,

$$K(s) \stackrel{s}{=} \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & 0 \end{array}\right]. \tag{2}$$

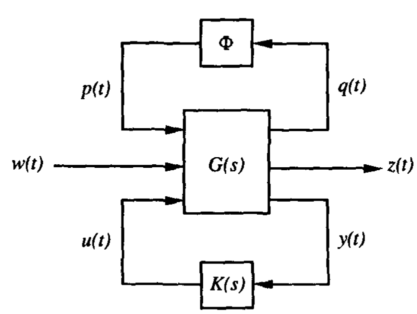

Fig. 1: Set-up for system analysis and synthesis.

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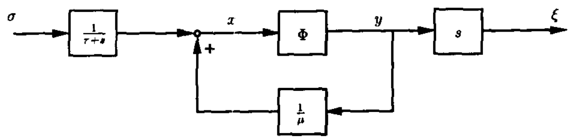

Fig. 2: Block diagram of *passive* operator $\tilde{\Phi} : \sigma \to \xi$

We present a technique that solves for the optimal controller by iteratively solving a set of linear matrix inequalities, with each iteration yielding a controller K that optimizes performance and an updated multiplier that guarantees stability. The performance we consider in is the $\mathcal{L}_2$ gain (or energy gain) of the closed loop system from w to z. In the remainder of the paper we first give some notation and theorems in section 2. In section 3 we review some previous analysis for the stability of systems with hysteresis nonlinearities, and extend those results for systems with an invertible system matrix. In section 4 we present the control design algorithm using the analysis and mathematics from the previous sections. A robust $\mathcal{H}_\infty$ loop shaping problem is given in section 5 to illustrate the benefits of the design procedure.

2 Notation and Definitions

Terminology and definitions used in this paper are standard. For definitions of the vector space $\mathcal{L}_{2e}$, $\mathcal{L}_2$-stability, passive operators, *etc.*, see, for example, [4]. For a mathematical treatment of hysteresis operators, see [7], and [3] for properties of passive hysteresis and related stability proofs. In addition we assume that the maximum slope at any point on the hysteresis is μ (see [3] for more details).

Definition 2.1 *Given a hysteresis $\Phi_P : \mathcal{L}_{2e} \to \mathcal{L}_{2e}$, we define a new operator with characteristic given by the input-output relation $\tilde{\Phi} : \sigma \to \xi$ defined in Figure (2), with $\tau \geq 0$.*

Lemma 2.1 *If $\Phi_P : \mathcal{L}_{2e} \to \mathcal{L}_{2e}$ is a passive hysteresis, then $\tilde{\Phi} : \sigma \to \xi$ as given by Definition 2.1 is a passive operator.*

Proof: see [3]. □

3 Robustness Analysis of Systems with Hysteresis Nonlinearities

In [3], passivity theory was used to develop robust stability tests for systems with passive hysteresis nonlinearities. This was accomplished using a loop transformation which converted the hysteresis into a passive operator, per Lemma 2, and modified linear system G_{qp} as

$$\tilde{G}_{qp}(s) = (G_{qp}(s) + \frac{1}{\mu})\frac{\tau + s}{s}. \qquad (3)$$

$\mathcal{L}_2$ stability for the system is then guaranteed by requiring that $\tilde{G}_{qp}$ be strictly passive. For LTI systems this is equivalent to having the transfer function be strictly positive real, or equivalently, for some $\delta > 0$,

$$Re\{\tilde{G}_{qp}(j\omega)\} > \delta, \quad \forall \omega \geq 0. \qquad (4)$$

The stability guarantee is obtained essentially by finding the free parameters $\tau \geq 0, \delta > 0$ such that (4) holds. With $\tilde{G}_{qp}$ is given by

$$\tilde{G}_{qp}(s) \stackrel{s}{=} \left[\begin{array}{c|c} \tilde{A} & \tilde{B}_p \\ \hline \tilde{C}_{q1} + \tau\tilde{C}_{q2} & \tilde{D}_{qp} \end{array}\right], \qquad (5)$$

where

$$\begin{array}{ll} \tilde{A} = \begin{bmatrix} A & 0 \\ C_q & 0 \end{bmatrix} & \tilde{B}_p = \begin{bmatrix} B_p \\ D_{qp} + 1/\mu \end{bmatrix} \\ \tilde{C}_{q1} = \begin{bmatrix} C_q & 0 \end{bmatrix} & \tilde{D}_{qp} = [D_{qp} + 1/\mu]. \\ \tilde{C}_{q2} = \begin{bmatrix} 0 & 1 \end{bmatrix} & \end{array} \qquad (6)$$

the stability condition (4) (strict passivity) is achieved by satisfying the set of matrix inequalities:

$$\delta > 0, \tau \geq 0, P = P^T > 0$$
$$\begin{bmatrix} -\tilde{A}^T P - P\tilde{A} & (\tilde{C}_{q1} + \tau\tilde{C}_{q2})^T - P\tilde{B}_p \\ (\tilde{C}_{q1} + \tau\tilde{C}_{q2}) - \tilde{B}_p^T P & \tilde{D}_{qp} + \tilde{D}_{qp}^T - 2\delta I \end{bmatrix} \geq 0. \qquad (7)$$

3.1 Decoupling the Constant Eigenspace

It is easy to show that when the original system $G(s)$ has a free integrator (i.e., zero eigenvalue) then it is necessary for the stability parameter $\tau = 0$ [3]. In this case the multiplier reduces to identity, and we are left with the positive real condition, $Re\{G(j\omega)\} > -1/\mu, \forall\omega > 0$ for the stability guarantee. If we discount the cases when G has a zero eigenvalue, we can simplify the stability test. This simplification is possible for these cases essentially because $|A| \neq 0$ and hence, A^{-1} exists and as a result, the system can be block diagonalized by changing coordinates with the transformation $z = Tx$, where

$$T = \begin{bmatrix} I & 0 \\ C_q A^{-1} & I \end{bmatrix}. \qquad (8)$$

This transformation decomposes the transfer function (3) into the constant and stable dynamic parts as

$$\tilde{G}_{qp}(s) = \tau\frac{R}{s} + G_r(s) \qquad (9)$$

where $R = G_{qp}(0) + \frac{1}{\mu}$ is the residue of the augmented system $\tilde{G}_{qp}$, and $G_{qp}(0) = -C_q A^{-1} B_p + D_{qp}$ the value of the original system (not augmented) at $s = 0$. The stable portion G_r is the augmented system reduced by the integrator state, given by the state space realization

$$G_r(s) \stackrel{s}{=} \left[\begin{array}{c|c} A & B_p \\ \hline C_q(I + \tau A^{-1}) & D_{qp} + 1/\mu \end{array}\right]. \qquad (10)$$

Now, the strict passivity requirement (4) on $\tilde{G}_{qp}(s)$ is addressed with the following proposition, with reference to the decoupled form (9).

Proposition 3.1 *(Strict Passivity) If there exists $\tau \geq 0, \delta > 0$ such that the following two conditions*

1. $R = G_{qp}(0) + \frac{1}{\mu} \geq 0$
2. *The reduced system G_r, given by (10) is dissipative with respect to the supply rate:*

$$r(p,q) = p^T q - \delta p^T p, \quad (11)$$

are satisfied, then the system $\tilde{G}_{qp}$ is strictly passive.

Proof: Let $\xi : R_+ \to R$ represent the integrator state with $\xi(0) = \xi_0$, $V : R^n \to R_+$ be a storage function for G_r and q_I, q_r be the outputs of the integrator and G_r, respectively. Then for any $T \geq 0$ we have

$$\begin{aligned}\int_0^T q^T p dt &= \int_0^T (q_I + q_r)^T p\, dt \\ &= \tau R \int_0^T \xi(t) \frac{d}{dt}\xi(t)\, dt + \int_0^T q_r^T p\, dt \\ &\geq \frac{\tau R}{2}(\xi_T^2 - \xi_0^2) + V(x_T) - V(x_0) + \delta \|p\|_T^2 \\ &\geq -\beta(\xi_0, x_0) + \delta\|p\|_T^2,\end{aligned}$$

where $\beta(\xi_0, x_0) = \tau R \xi_0^2/2 + V(x_0) \geq 0$, and thus, $\tilde{G}_{qp}$ is strictly passive by the definition given in [4]. □
Decomposing the system in this way allows us to establish the stability requirement in two parts. Condition 1 ensures the passivity (nonstrict) of the pure integrator, while condition 2 enforces the strict passivity of the augmented system $\tilde{G}_{qp}$ by guaranteeing a certain dissipation rate, with $\delta > 0$. It is straightforward to show that extending this approach to the case of multiple nonlinearities would necessarily require $R = R^T \geq 0$. Condition 2 is naturally expressed in an LMI in the reduced system (10) matrices as given by the following corollary.

Corollary 3.1 *(Strict Passivity LMI) With the storage function $V = x^T P x, P = P^T > 0, x \in R^n$ being the state of the linear system G_r, Condition 2. in Proposition 3.1 is equivalent to the feasibility of the LMI*

$$\begin{bmatrix} -A^T P - PA & (I + \tau A^{-1})^T C_q^T - PB_p \\ (\cdot)_{12}^T & 2(D_{qp} + 1/\mu) - 2\delta \end{bmatrix} > 0. \quad (12)$$

Proof: Use definition of dissipation with respect to the supply rate (11) for the system G_r having the given storage function. For details and definitions, see [8]. □
For a linear system to be strictly positive real, the feedthrough term must be positive in order to guarantee positivity in the limit as $s \to \infty$. This requirement is apparent in the $(2,2)$ element of LMI (12). The required positivity as $s \to 0$ is ensured by $R > 0$, as given by Condition 1. of the above proposition. These two limit requirements and the behavior at all finite ω can be summarized by the following frequency domain test.

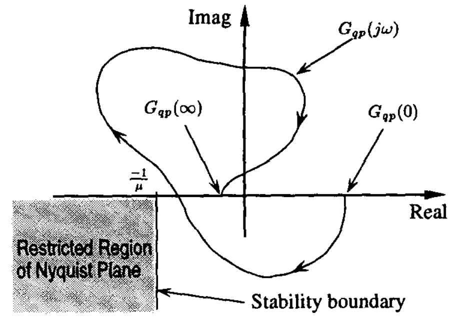

Fig. 3: G_{qp} satisfies residue condition, $G_{qp}(0) > -1/\mu$ and positivity requirement $G_{qp}(\infty) > -1/\mu$.

Corollary 3.2 *(Strict Passivity Nyquist Test) There will exist $\delta > 0, \tau \geq 0$ satisfying the conditions of Proposition 3.1 if the Nyquist of $G_{qp}(j\omega)$ never enters (or touches) the orthant in the complex plane to the left of the $-1/\mu$ on the real axis, as depicted in Figure 3.*

Proof: The proof of this assertion is omitted for space considerations. However, one simple proof proceeds by construction, simply by breaking up the Nyquist plane into the three allowed regions and finding the minimum $\tau \geq 0$ that will satisfy the regions simultaneously. □
In practice, Corollary 3.2 can be used to graphically test for stability prior to any system augmentation or LMI computations. If the system G_{qp} passes the Nyquist test, then the LMI defined in Corollary 3.1 can be solved to yield the storage function and stability parameters.

3.2 Robust Performance

The robust stability analysis presented thus far is extended to include a metric on the performance channel from w to z. Using an upper bound for $\mathcal{L}_2$-gain as the criteria, this extended problem can be formulated by requiring the closed loop system, Figure (1), be dissipative with respect to the supply rate:

$$r(p,q,w,z) = \gamma^2 w^T w - z^T z + \lambda p^T q - \delta p^T p,$$

as is shown in [3]. We can simultaneously satisfy a norm bound constraint on the performance while guaranteeing $\mathcal{L}_2$-stability by solving:

$$\begin{aligned} \text{minimize} \quad & \gamma^2 \\ \text{subject to:} \quad & \lambda, \tau, \delta > 0 \\ & M \geq 0, P > 0 \end{aligned} \quad (13)$$

where M is a block-3 × 3 matrix with entries $-M_{11} = \tilde{A}^T P + P\tilde{A} + \tilde{C}_z^T \tilde{C}_z$, $M_{12} = \lambda \tilde{C}_{q1}^T + \tau \tilde{C}_{q2}^T - P\tilde{B}_p - \tilde{C}_z^T \tilde{D}_{zp}$, $M_{13} = -P\tilde{B}_w - \tilde{C}_z^T \tilde{D}_{zw}$, $M_{22} = \lambda(\tilde{D}_{qp} + \tilde{D}_{qp}^T) - 2\delta I - \tilde{D}_{zp}^T \tilde{D}_{zp}$, $M_{23} = \lambda \tilde{D}_{qw} - \tilde{D}_{zp}^T \tilde{D}_{zw}$, and $M_{33} = \gamma^2 - \tilde{D}_{zw}^T \tilde{D}_{zw}$. This optimization problem (13) is the basis for the robust $\mathcal{H}_\infty$ synthesis, presented next.

4 Controller Design

This section describes a method to synthesize robust controllers for systems containing hysteresis nonlinearities. The synthesis technique developed here uses an LMI framework to develop controllers which solve the optimization problem (13). Because the controller optimization problem is bilinear in the controller and multiplier variables (and thus is not an LMI), the solution is iterative and consists of 3 main steps: controller optimization, a controller reconstruction and a multiplier update, each of which is an LMI. This iterative LMI approach to solving the BMI problem is known to be nonconvex and thus not guaranteed to converge to the global minimum. Nevertheless it has been shown to be an effective means of designing robust controllers [5]. Note that the same algorithm will apply for both the system fully augmented with the multiplier, as given in (5)–(6) and the reduced system with the multiplier states removed (10). The main difference is that for the full system

$$C_{q1} = [\ C_q \mid 0\ ],\quad C_{q2} = [\ 0 \mid I\ ]$$

but the reduced system has $C_{q1} = C_q$ and $C_{q2} = C_q A^{-1}$.

4.1 Controller Elimination

To simplify the design process the variable A_c is eliminated from the optimization problem. This simplification reduces the number of variables in the optimization and leads to smaller LMI's. To proceed, define

$$\tilde{A}_0 := \begin{bmatrix} A & B_u C_c \\ B_c C_y & B_c D_{yu} C_c \end{bmatrix},\quad \tilde{J} := \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{14}$$

Then $\tilde{A} = \tilde{A}_0 + \tilde{J} A_c \tilde{J}^T$ and the constraint $M > 0$ is

$$M = M_0 - V A_c^T U^T - U A_c V^T > 0, \tag{15}$$

where M_0 is the same matrix as that in (13) with A_c eliminated, and U, V are defined as

$$U^T := [\ \tilde{J}^T \tilde{P}\quad 0\quad 0\ ],\quad V^T := [\ \tilde{J}^T\quad 0\quad 0\ ].$$

By the Elimination Lemma [9, pp. 32–33] it can be shown that the constraint $M > 0$ holds if and only if

$$V_\perp^T M_0 V_\perp > 0,\quad U_\perp^T M_0 U_\perp > 0 \tag{16}$$

are satisfied, where $U_\perp$ and $V_\perp$ are the orthogonal complements of U and V, respectively. This is a standard technique of ridding A_c from the optimization problem. Proceeding, we partition $\tilde{P}$ and its inverse $\tilde{Q}$ as

$$\tilde{P} = \begin{bmatrix} P & M \\ M^T & R \end{bmatrix},\quad \tilde{Q} = \tilde{P}^{-1} = \begin{bmatrix} Q & N \\ N^T & S \end{bmatrix}, \tag{17}$$

where P and $Q \in \mathbf{R}^{n\times n}$. N is related to P, Q, and M in the form satisfying $N = (I - QP)M^{-T}$. We define $Y := C_c N^T$ and $Z := MB_c$. Then, with algebraic simplification, the constraints (16) become

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{12}^T & F_{22} & F_{23} \\ F_{13}^T & F_{23}^T & F_{33} \end{bmatrix} > 0,\quad \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{12}^T & H_{22} & H_{23} & 0 \\ H_{13}^T & H_{23}^T & H_{33} & 0 \\ H_{14}^T & 0 & 0 & I \end{bmatrix} > 0 \tag{18}$$

where the matrices for the first constraint in (18) are:

$$\begin{aligned}
F_{11} &= -A^T P - PA - ZC_y - C_y^T Z - C_z^T C_z \\
F_{13} &= -PB_w - ZD_{yw} - C_z^T D_{zw} \\
F_{12} &= -PB_p - C_z^T D_{zp} - ZD_{yp} + \lambda C_{q1}^T + \tau C_{q2}^T \\
F_{22} &= \lambda(D_{qp} + D_{qp}^T) - 2\delta I - D_{zp}^T D_{zp} \\
F_{23} &= \lambda D_{qw} - D_{zp}^T D_{zw} \\
F_{33} &= \gamma^2 I - D_{zw}^T D_{zw}.
\end{aligned}$$

A similar set of matrices in Q can be defined for the second inequality (see [5] for more details). The entries in the second constraint involve terms that contain products of the multiplier parameters λ, and τ and matrix variables Q and Y. Thus this constraint is not a linear matrix inequality (LMI), but a bilinear matrix inequality (BMI). The final constraint that Lyapunov matrices $\tilde{P}, \tilde{Q} > 0$, as parameterized in the Completion Lemma [10], is implied by the existence of symmetric matrices W and Z such that

$$\begin{bmatrix} X & Z^T & 0 & 0 \\ Z & P & I & 0 \\ 0 & I & Q & Y^T \\ 0 & 0 & Y & W \end{bmatrix} > 0. \tag{19}$$

Summarizing, then, in eliminating A_c from problem (13) we derive the equivalent problem,

$$\begin{aligned} \text{minimize}\quad & \gamma^2 \\ \text{subject to:}\quad & \lambda, \tau \geq 0,\quad \delta > 0 \\ & (18),\ (19) \end{aligned} \tag{20}$$

which involves constraints that are bilinear in the multiplier parameters and matrix variables.

4.2 Controller Reconstruction

Finding a controller $K(s)$ with the state space parameterization (A_c, B_c, C_c) given a set of variables $(P, Q, W, X, Y, Z, \gamma, \lambda, \tau, \delta)$ satisfying (18), (19) is referred to as controller reconstruction. This requires first the formation of the quadratic term of the Lyapunov function $\tilde{P}$ (as defined in [11]) and then solving the feasibility LMI (15) for some A_c. Then the controller input and output matrices are given respectively by $B_c = M^{-1}Z$ and $C_c = Y(I - PQ)^{-1}M$. Where M here is an arbitrary nonsingular matrix used to parameterize $\tilde{P}$.

Table 1: Controller synthesis algorithm

1. Initialize stability parameters (λ, τ, δ) with feasible controller.
2. Repeat until $\Delta\gamma < \epsilon$
   (a) Controller optimization: solve (20)
   (b) Reconstruct $A_{c,k}$: solve (15)
   (c) Update stability parameters: solve (13)
   (d) Compute change: $\Delta\gamma = |\gamma_k - \gamma_{k-1}|$

4.3 Synthesis Algorithm

The design algorithm is initialized with a feasible controller by some appropriate means of controller synthesis. In this case feasibility of initial robust controller means that Nyquist plot satisfies the conditions of Corollary 3.2. With proper initialization, the control design then proceeds as summarized in the Table 1. The algorithm repeats until the performance converges to a value which, as noted, may not be a global minimum.

5 Numerical Example: Robust Loop Shaping

In this section we consider the use of the robust synthesis method described in section 4 to design a controller that optimizes a S/KS mixed sensitivity cost objective [12, pages 369–375]. The set-up for the synthesis is shown in Figure 4, where the hysteresis $\Phi : q \to p$ is taken to be passive, with maximum slope $\mu = 1$. The plant

$$G_1(s) = \frac{7.5(s^2 - 0.2s + 0.1)}{s^3 + 2s^2 + 2s + 1},$$

has a pair of nonminimum phase zeros at $0.1 \pm 0.3j$. The performance weights, given as

$$W_1(s) = \frac{350s + 28}{500s + 1}, \quad W_2(s) = \frac{150s + 75}{s + 100},$$

are used to frequency weight the loop transfer functions. The objective of S/KS mixed sensitivity is to design a controller $K(s)$ minimize the cost objective

$$\gamma = \left\| \begin{bmatrix} W_1 S \\ W_2 KS \end{bmatrix} \right\|_\infty,$$

where the sensitivity function, $S = (I + KG)^{-1}$, is a measure of the closed loop disturbance rejection. By minimizing γ, we simultaneously minimize the peak gain in the error to a disturbance w and limit the bandwidth of the controller. Since $G_{qp}(0) = 0.75$ we have that $G_{qp}(0) > -1/\mu$ and since the system has no zero eigenvalues, we can use the parameterization of the system given in section 3.1, and design a controller K of order equal to that of the original plant with the augmented weights, that is $A_k \in \mathbf{R}^{5\times5}$. To initialize the synthesis algorithm, we designed a controller that optimizes a skewed-μ, or μ^s metric [12, page 321]. This controller gives the best $\mathcal{H}_\infty$ performance subject to a stability guarantee for the system with this nonlinearity. Stability is guaranteed by maintaining a norm bound constraint on the robustness channel: $|G_{qp}(j\omega)| < 1$, $\forall\omega \geq 0$. This controller serves as a good initial condition for our synthesis algorithm since the norm bound means that $G_{qp}(j\omega)$ satisfies Corollary 3.2. The resulting closed loop transfer function for the system with the μ^s-design is shown in a Nyquist plot, Figure 5. Since $\mu^s(\tilde{G}) = 0.95$ as expected, the graph of $G_{qp}(j\omega)$ stays within the unit circle. However, the performance of the μ-design is not very good. As shown in Figure 7, both the sensitivity S and the KS objective exceed the desired

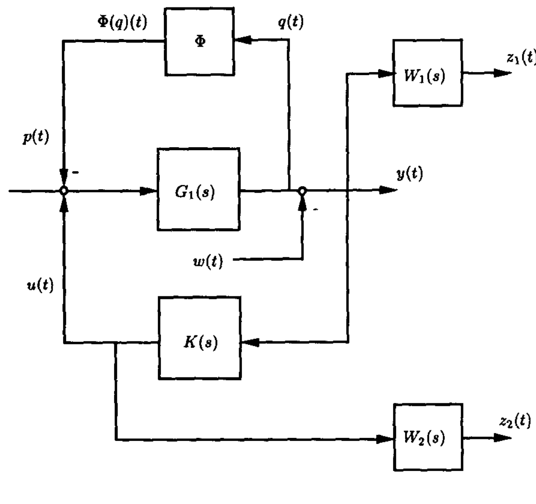

Fig. 4: S/KS mixed sensitivity synthesis set up.

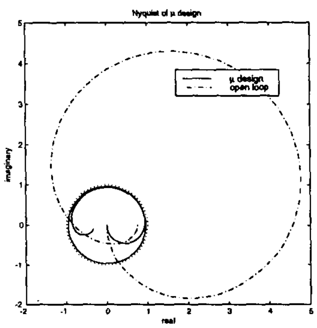

Fig. 5: Nyquist for μ-design

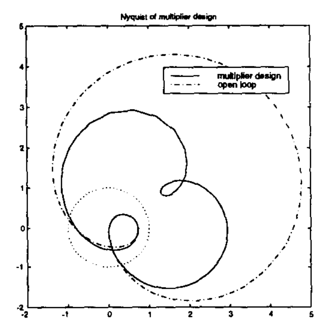

Fig. 6: Nyq. for mult. ctl.

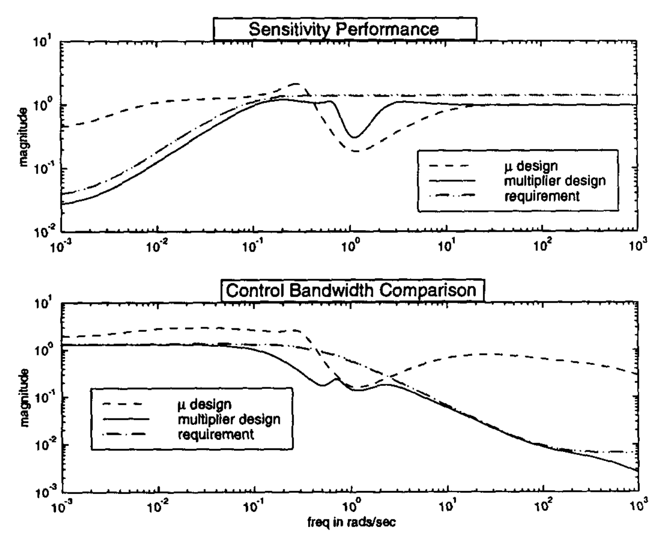

Fig. 7: Performance curves for μ and multiplier controllers

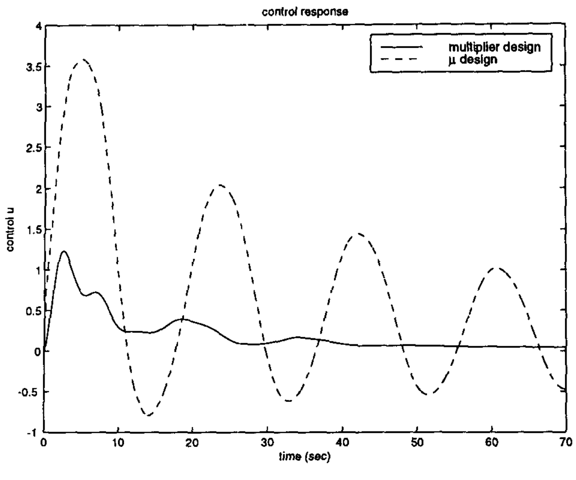

Fig. 8: Comparison of control responses, $u(t)$.

loop shaping requirements. In low frequency, the sensitivity requirement is violated by factor of 10 while the KS criteria exceeds the desired gain by nearly a factor of 25. Using the μ controller as the initial condition, the synthesis algorithm ran 18 iterations before converging to a new controller, reaching the stopping criteria set by $\epsilon = 0.0005$. During the iteration, the performance improved by a factor of 50, with γ reducing from 0.75 down to a value less than 0.015. As a result, the S/KS performance objective is satisfied, as indicated in Figure 7, since the graphs of the unweighted S and KS functions lie under the required shaping curves, W_1^{-1} and W_2^{-1}. Stability is guaranteed since G_{qp} satisfies the condition of Corollary 3.2, as indicated in Figure 6. Comparing the Nyquist and Bode plots, it is clear that the robustness and performance requirements are competing. That is, in maintaining G_{qp} inside the unit circle, the μ-controller sacrificed a great deal of performance. The multiplier controller, however, takes advantage of the less restrictive stability requirement in order to satisfy the performance objectives. A simulation of the nonlinear system confirms the improved performance with the multiplier controller. Included in the simulation was a Preisach hysteresis [7] with unity maximum slope. The control response $u(t)$ to a particular disturbance $w(t) \in \mathcal{L}_2$ for the closed loop system using the $\mu-$controller (dashed) and multiplier controller (solid) is displayed in Figure 8. The multiplier controller response peaks at a value of only one third of that for the $\mu-$controller and dies out after 40 seconds, while the $\mu-$control still oscillates with large amplitude after 70 seconds. These results provide strong confirmation that this new algorithm is an effective means for designing robust $\mathcal{H}_\infty$ controllers for systems with hysteresis.

6 Conclusions

In this paper we present a new design technique for robust control of systems having hysteresis nonlinearities using an LMI framework. Additionally, we extend recently developed stability criteria with a method of removing the constant eigenspace associated with the multiplier used in the analysis procedure. This allows the direct use of commonly available interior point methods to solve for the optimal controllers without having to solve an approximate problem in which the constant eigenspace is replaced with a stable one. The extension provides the additional advantage of allowing for reduced order controller design in the sense that the resulting controllers do not require additional states associated with the multiplier. Thus, there is no penalty for the stability guarantee. The algorithm presented designs robust $\mathcal{H}_\infty$ controllers by iteratively solving a set of LMI's . The effectiveness of the new design technique is illustrated in solving a loop shaping control problem for a non-minimum phase system.

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