Anti-windup Compensation for Nonlinear Systems via Gradient Projection

Application to Adaptive Control

Justin Teo and Jonathan P. How

Aerospace Controls Laboratory
Department of Aeronautics & Astronautics
Massachusetts Institute of Technology

CDC/CCC '09
December 18, 2009

Outline

Motivation

Some well recognized facts (Khalil 2002, Bernstein and Michel 1995)

Most practical control systems

- are **nonlinear**, eg. Euler-Lagrange systems, deadzone, backlash, hysteresis, driven by **nonlinear** controllers
- have **actuation limits**, eg. deflection & thrust limits in aircrafts, acceleration/deceleration & steering limits in cars, cooling capacity in air-conditioners

Effects called "windup" (Tarbouriech and Turner 2009)

When system driven to saturation limits

- performance degradation (with **certainty**)
- destabilize (possibly)

Motivation

Some well recognized facts (Khalil 2002, Bernstein and Michel 1995)

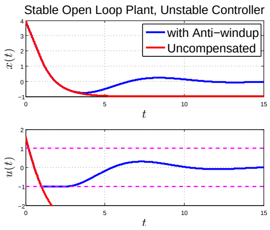
Most practical control systems

- are **nonlinear**, eg. Euler-Lagrange systems, deadzone, backlash, hysteresis, driven by **nonlinear** controllers
- have **actuation limits**, eg. deflection & thrust limits in aircrafts, acceleration/deceleration & steering limits in cars, cooling capacity in air-conditioners

Effects called "windup" (Tarbouriech and Turner 2009)

When system driven to saturation limits

- performance degradation (with **certainty**)
- destabilize (possibly)



Prior Work

Anti-windup compensation for linear time invariant (LTI) systems well developed. Surveys: (Tarbouriech and Turner 2009, Kothare et al. 1994, Edwards and Postlethwaite 1998)

Anti-windup for **nonlinear** systems:

- feedback linearizable systems: (Calvet and Arkun 1988, Kendi and Doyle 1997, Kapoor and Daoutidis 1997, 1999, Herrmann et al. 2006, Menon et al. 2006, 2008b,a, Yoon et al. 2008)
- with **specific** adaptive controllers: (Hu and Rangaiah 2000, Johnson and Calise 2003, Kahveci et al. 2007)
- for Euler-Lagrange systems: (Morabito et al. 2004)

Open Problem (Tarbouriech and Turner 2009)

Anti-windup compensation for nonlinear systems/controllers

Prior Work

Anti-windup compensation for linear time invariant (LTI) systems well developed. Surveys: (Tarbouriech and Turner 2009, Kothare et al. 1994, Edwards and Postlethwaite 1998)

Anti-windup for **nonlinear** systems:

- feedback linearizable systems: (Calvet and Arkun 1988, Kendi and Doyle 1997, Kapoor and Daoutidis 1997, 1999, Herrmann et al. 2006, Menon et al. 2006, 2008b,a, Yoon et al. 2008)
- with **specific** adaptive controllers: (Hu and Rangaiah 2000, Johnson and Calise 2003, Kahveci et al. 2007)
- for Euler-Lagrange systems: (Morabito et al. 2004)

Open Problem (Tarbouriech and Turner 2009)

Anti-windup compensation for nonlinear systems/controllers

Prior Work

Anti-windup compensation for linear time invariant (LTI) systems well developed. Surveys: (Tarbouriech and Turner 2009, Kothare et al. 1994, Edwards and Postlethwaite 1998)

Anti-windup for **nonlinear** systems:

- feedback linearizable systems: (Calvet and Arkun 1988, Kendi and Doyle 1997, Kapoor and Daoutidis 1997, 1999, Herrmann et al. 2006, Menon et al. 2006, 2008b,a, Yoon et al. 2008)
- with **specific** adaptive controllers: (Hu and Rangaiah 2000, Johnson and Calise 2003, Kahveci et al. 2007)
- for Euler-Lagrange systems: (Morabito et al. 2004)

Open Problem (Tarbouriech and Turner 2009)

Anti-windup compensation for nonlinear systems/controllers

Overview

Gradient Projection Anti-windup (GPAW) scheme:

- generalization of "stop integration" heuristic (Åström and Rundqwist 1989) to multi-input-multi-output (MIMO) nonlinear systems/controllers, ie. general purpose
- requires solving a **combinatorial** optimization sub-problem
- attempts to maintain controller state-output consistency

To develop GPAW scheme, need to:

- extend gradient projection method of nonlinear programming (Rosen 1960, 1961) to **continuous-time**
- use continuous-time gradient projection (only) to project **controller state** to unsaturated region

Note: Last idea well known in adaptive control to bound parameter estimates in some a priori known region (Ioannou and Sun 1996, Pomet and Praly 1992), but only for **single** nonlinear constraint

Overview

Gradient Projection Anti-windup (GPAW) scheme:

- generalization of “stop integration” heuristic (Åström and Rundqwist 1989) to multi-input-multi-output (MIMO) nonlinear systems/controllers, ie. general purpose
- requires solving a **combinatorial** optimization sub-problem
- attempts to maintain controller state-output consistency

To develop GPAW scheme, need to:

- extend gradient projection method of nonlinear programming (Rosen 1960, 1961) to **continuous-time**
- use continuous-time gradient projection (only) to project **controller state** to unsaturated region

Note: Last idea well known in adaptive control to bound parameter estimates in some a priori known region (Ioannou and Sun 1996, Pomet and Praly 1992), but only for **single** nonlinear constraint

Overview

Gradient Projection Anti-windup (GPAW) scheme:

- generalization of “stop integration” heuristic (Åström and Rundqwist 1989) to multi-input-multi-output (MIMO) nonlinear systems/controllers, ie. general purpose
- requires solving a **combinatorial** optimization sub-problem
- attempts to maintain controller state-output consistency

To develop GPAW scheme, need to:

- extend gradient projection method of nonlinear programming (Rosen 1960, 1961) to **continuous-time**
- use continuous-time gradient projection (only) to project **controller state** to unsaturated region

Note: Last idea well known in adaptive control to bound parameter estimates in some a priori known region (Ioannou and Sun 1996, Pomet and Praly 1992), but only for **single** nonlinear constraint

Problem Statement

Given input constrained plant and **nominal** controller

$$\Sigma_{sys}: \begin{cases} \dot{x} = f(x, \text{sat}(u)), \\ y = g(x, \text{sat}(u)), \end{cases} \quad \Sigma_{cont}: \begin{cases} \dot{x}_c = f_c(x_c, y, r(t)), \quad x_c(0) = x_{co}, \\ u = g_c(x_c, y, r(t)), \end{cases}$$

design **anti-windup compensated** controller

$$\Sigma_{aw\_cont}: \begin{cases} \dot{x}_g = f_g(x_g, y, r(t)), \quad x_g(0) = x_{c0}, \\ u_g = g_g(x_g, y, r(t)), \end{cases}$$

so **nominal uncompensated** system Σ_n (feedback interconnection (FI) of Σ_{sys} and Σ_{cont}) and **anti-windup compensated** system Σ_g (FI of Σ_{sys} and Σ_{aw_cont} with $u := u_g$) satisfy

1. when no controls saturate, nominal performance recovered, ie. $u_g \equiv g_c(x_c, y, r(t))$
2. when some controls saturate, performance of Σ_g is **no worse than** Σ_n, and performance of Σ_g **degrades gracefully** with severity of saturation constraints

Problem Statement

Given input constrained plant and **nominal** controller

$$\Sigma_{sys}: \begin{cases} \dot{x} = f(x, \mathrm{sat}(u)), \\ y = g(x, \mathrm{sat}(u)), \end{cases} \quad \Sigma_{cont}: \begin{cases} \dot{x}_c = f_c(x_c, y, r(t)), \quad x_c(0) = x_{co}, \\ u = g_c(x_c, y, r(t)), \end{cases}$$

design **anti-windup compensated** controller

$$\Sigma_{aw\_cont}: \begin{cases} \dot{x}_g = f_g(x_g, y, r(t)), \quad x_g(0) = x_{c0}, \\ u_g = g_g(x_g, y, r(t)), \end{cases}$$

so **nominal uncompensated** system Σ_n (feedback interconnection (FI) of Σ_{sys} and Σ_{cont}) and **anti-windup compensated** system Σ_g (FI of Σ_{sys} and Σ_{aw_cont} with $u := u_g$) satisfy

1. when no controls saturate, nominal performance recovered, ie. $u_g \equiv g_c(x_c, y, r(t))$
2. when some controls saturate, performance of Σ_g is **no worse than** Σ_n, and performance of Σ_g **degrades gracefully** with severity of saturation constraints

Problem Statement

Given input constrained plant and **nominal** controller

$$\Sigma_{sys}: \begin{cases} \dot{x} = f(x, \mathrm{sat}(u)), \\ y = g(x, \mathrm{sat}(u)), \end{cases} \quad \Sigma_{cont}: \begin{cases} \dot{x}_c = f_c(x_c, y, r(t)), \quad x_c(0) = x_{co}, \\ u = g_c(x_c, y, r(t)), \end{cases}$$

design **anti-windup compensated** controller

$$\Sigma_{aw\_cont}: \begin{cases} \dot{x}_g = f_g(x_g, y, r(t)), \quad x_g(0) = x_{c0}, \\ u_g = g_g(x_g, y, r(t)), \end{cases}$$

so **nominal uncompensated** system Σ_n (feedback interconnection (FI) of Σ_{sys} and Σ_{cont}) and **anti-windup compensated** system Σ_g (FI of Σ_{sys} and Σ_{aw_cont} with $u := u_g$) satisfy

1. when no controls saturate, nominal performance recovered, ie. $u_g \equiv g_c(x_c, y, r(t))$
2. when some controls saturate, performance of Σ_g is **no worse than** Σ_n, and performance of Σ_g **degrades gracefully** with severity of saturation constraints

Intuition from Decoupled Nonlinear MIMO Systems

"Conditionally Freeze Integrator" method (Hodel and Hall 2001):

$$\dot{e}_i = e, \quad u = K_p e + K_i e_i, \quad \xrightarrow{\text{anti-windup}} \quad \dot{e}_i = \begin{cases} 0, & \text{if } ((e > 0) \wedge (u \geq u_{max})), \\ 0, & \text{if } ((e < 0) \wedge (u \leq u_{min})), \\ e, & \text{otherwise.} \end{cases} \quad u = K_p e + K_i e_i.$$

Extend to **decoupled** nonlinear MIMO controllers:

$$\dot{x}_i = f_i(x_i, y_i, r_i), \quad u_i = g_i(x_i, y_i, r_i), \quad \xrightarrow{\text{anti-windup}} \quad \dot{x}_i = \begin{cases} 0, & \text{if } A, \\ f_i(x_i, y_i, r_i), & \text{otherwise.} \end{cases} \quad u_i = g_i(x_i, y_i, r_i),$$

where logical statement A suppresses "undesirable" updates

$$A = \left((u_i \geq u_{imax}) \wedge \left(\frac{\partial g_i}{\partial x_i} f_i(x_i, y_i, r_i) > 0 \right) \right) \vee \left((u_i \leq u_{imin}) \wedge \left(\frac{\partial g_i}{\partial x_i} f_i(x_i, y_i, r_i) < 0 \right) \right)$$

Intuition from Decoupled Nonlinear MIMO Systems

"Conditionally Freeze Integrator" method (Hodel and Hall 2001):

$$\dot{e}_i = e, \quad u = K_p e + K_i e_i, \quad \xrightarrow{\text{anti-windup}} \quad \dot{e}_i = \begin{cases} 0, & \text{if } ((e > 0) \wedge (u \geq u_{max})), \\ 0, & \text{if } ((e < 0) \wedge (u \leq u_{min})), \\ e, & \text{otherwise.} \end{cases} \quad u = K_p e + K_i e_i.$$

Extend to **decoupled** nonlinear MIMO controllers:

$$\dot{x}_i = f_i(x_i, y_i, r_i), \quad u_i = g_i(x_i, y_i, r_i), \quad \xrightarrow{\text{anti-windup}} \quad \dot{x}_i = \begin{cases} 0, & \text{if } A, \\ f_i(x_i, y_i, r_i), & \text{otherwise.} \end{cases} \quad u_i = g_i(x_i, y_i, r_i),$$

where logical statement A suppresses "undesirable" updates

$$A = \left((u_i \geq u_{imax}) \wedge \left(\frac{\partial g_i}{\partial x_i} f_i(x_i, y_i, r_i) > 0 \right) \right) \vee \left((u_i \leq u_{imin}) \wedge \left(\frac{\partial g_i}{\partial x_i} f_i(x_i, y_i, r_i) < 0 \right) \right)$$

Gradient Projection Method I

- To extend to **coupled** nonlinear MIMO systems/controllers, **update controller state vector in nominal direction as much as possible, while attempting not to aggravate existing saturation constraints** $\Rightarrow$ **gradient projection**

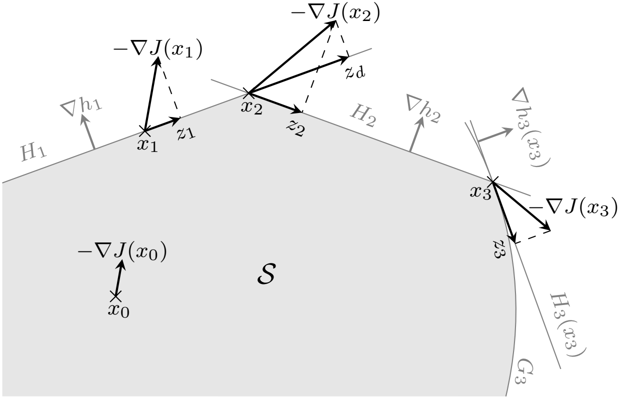
- Gradient Projection Method (Rosen 1960, 1961) solves constrained nonlinear programs

$$\min_{x \in \mathbb{R}^q} J(x), \qquad \text{subject to } h(x) \leq 0 \in \mathbb{R}^k.$$

- Reduces to steepest descent method in the absence of active constraints

- Maintains feasibility by projection of nominal descent direction along **multiple** gradient vectors of active constraints

Gradient Projection Method II



Feasible region: $\mathcal{S} = \{x \in \mathbb{R}^q \mid h(x) \leq 0\}$

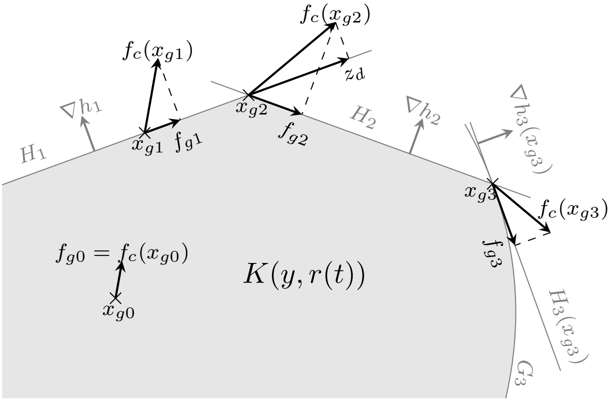
Gradient Projection Anti-windup Scheme

- Can extend Gradient Projection Method to **continuous-time**, similar to (Ioannou and Sun 1996) for a **single** nonlinear constraint
- Continuous-time Gradient Projection Method requires solving a **combinatorial** optimization subproblem online
- Use **only** gradient projection part to construct GPAW controller, with constraints defined by $2m$ saturation limits

$$h_i(x_g) := g_{ci}(x_g, y, r(t)) - u_{imax} \leq 0, \quad \forall i \in \{1, 2, \ldots, m\}$$
$$h_{i+m}(x_g) := -g_{ci}(x_g, y, r(t)) + u_{imin} \leq 0, \quad \forall i \in \{1, 2, \ldots, m\}.$$

- GPAW scheme has a **single** tuning parameter, $\Gamma \in \mathbb{R}^{q \times q}$, symmetric positive definite

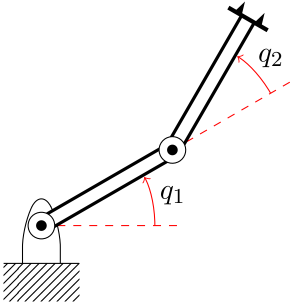
GPAW Scheme Visualization



- Note: $f_c(x_{gi}) := f_c(x_{gi}, y, r(t))$ and $f_{gi} := f_g(x_{gi}, y, r(t))$
- Unsaturated region is $K(y, r(t)) = \{x \in \mathbb{R}^q \mid \text{sat}(g_c(x, y, r(t))) = g_c(x, y, r(t))\}$

Application to Adaptive Sliding Mode Control

- Consider robot manipulator described by **nonlinear** Euler-Lagrange equations
- In the **absence** of actuator limits, an adaptive sliding mode controller (Slotine and Coetsee 1986) achieves GAS
- With actuator limits, apply GPAW scheme to obtain GPAW compensated controller



$$\dot{\hat{a}} = -\Phi Y^{\mathrm{T}} s, \qquad u = Y\hat{a} - K_d s, \qquad \xrightarrow{\text{GPAW}}$$

$$\dot{\hat{a}} = \begin{cases} -\Phi Y^{\mathrm{T}} s, & \text{if } A, \\ -\left(I - \frac{1}{y_1 \Gamma y_1^{\mathrm{T}}} \Gamma y_1^{\mathrm{T}} y_1\right) \Phi Y^{\mathrm{T}} s, & \text{if } B, \\ -\left(I - \frac{1}{y_2 \Gamma y_2^{\mathrm{T}}} \Gamma y_2^{\mathrm{T}} y_2\right) \Phi Y^{\mathrm{T}} s, & \text{if } C, \\ -\left(I - \Gamma Y^{\mathrm{T}} \left(Y \Gamma Y^{\mathrm{T}}\right)^{-1} Y\right) \Phi Y^{\mathrm{T}} s, & \text{else}, \end{cases}$$

$$u = Y\hat{a} - K_d s.$$

Simulation Results

- Fix nominal controller gains, (Λ, K_d, Φ), sinusoidal reference and simulate 5 cases
  1. unconstrained nominal system
  2. constrained, uncompensated system
  3. constrained with "stop integration" rule

$$\dot{\hat{a}} = \begin{cases} -\Phi Y^{\mathrm{T}} s, & \text{if } \operatorname{sat}(u) = u, \\ 0, & \text{otherwise.} \end{cases}$$

  4. constrained with GPAW compensation, $\Gamma = \Phi$
  5. constrained with GPAW compensation, $\Gamma = I$

- Set $u_{lim} := u_{imax} = -u_{imin}$, simulate 6 scenarios $u_{lim} \in \{180, 150, 120, 90, 60, 30\}$ Nm

- Show movies

Simulation Results

- Fix nominal controller gains, (Λ, K_d, Φ), sinusoidal reference and simulate 5 cases
  1. unconstrained nominal system
  2. constrained, uncompensated system
  3. constrained with "stop integration" rule

$$\dot{\hat{a}} = \begin{cases} -\Phi Y^{\mathrm{T}} s, & \text{if } \operatorname{sat}(u) = u, \\ 0, & \text{otherwise.} \end{cases}$$

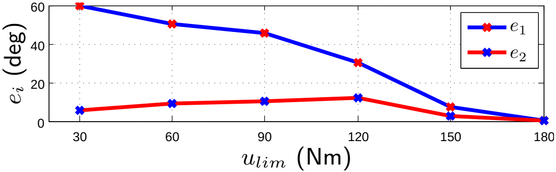
  4. constrained with GPAW compensation, $\Gamma = \Phi$
  5. constrained with GPAW compensation, $\Gamma = I$

- Set $u_{lim} := u_{imax} = -u_{imin}$, simulate 6 scenarios $u_{lim} \in \{180, 150, 120, 90, 60, 30\}$ Nm

- **Show movies**

Graceful Performance Degradation

- Let e_i be peak steady state tracking errors
- Graceful performance degradation for case 4 (GPAW with $\Gamma = \Phi$)



Limitations of GPAW Scheme

- Full controller state-output consistency achieved when $\mathrm{sat}(u) \equiv u$
- In general, GPAW scheme only achieves state-output consistency **approximately**
- Only controller state modified. Likely ineffective when

$$\left\| \frac{\partial g_c}{\partial x_c} \dot{x}_c \right\| \ll \left\| \frac{\partial g_c}{\partial y} \dot{y} + \frac{\partial g_c}{\partial r} \dot{r} \right\|$$

Conclusions

- Anti-windup compensation for nonlinear systems/controllers remains an **open problem** (Tarbouriech and Turner 2009)
- Extended gradient projection method (Rosen 1960, 1961) to continuous time
- Used gradient projection to construct GPAW compensated controller
- GPAW scheme
  - can be viewed as a generalization of "stop integration" heuristic
  - requires online solution to a **combinatorial** optimization subproblem
- Demonstrated viability of GPAW scheme on a non-trivial nonlinear system
- Identified some limitations
- Lots of interesting questions remain

Acknowledgments

- **Prof. Jean-Jacques Slotine** (MIT Mechanical Engineering Department, Nonlinear Systems Laboratory) for critical insights
- Dr. Han-Lim Choi (MIT Aeronautics & Astronautics Department, Aerospace Controls Laboratory) for insights into combinatorial optimization subproblem
- Funding agencies:
  - DSO National Laboratories, Singapore
  - Air Force Office of Scientific Research (AFOSR), USA

Questions

Questions?

Recent Results

- When restricted to 1st-order constrained LTI system driven by 1st-order LTI controller, can show
  - GPAW compensated system is a **projected dynamical system** (PDS) (Dupuis and Nagurney 1993, Zhang and Nagurney 1995, Nagurney and Zhang 1996, Cojocaru and Jonker 2004) (PDS is an independent line of research that has attracted significant attention of economists, physicists and mathematicians)
  - GPAW scheme can only **maintain/enlarge exact** region of attraction
- When output equation of nominal controller depends **only on controller state**, $u = g_c(x_c)$, ie. not on measurements and/or exogenous inputs, then **exact** state-output consistency achieved when appropriately initialized - **eliminates** previously identified limitations
- Under similar conditions, derived geometric bounding condition foreseen to aid in Lyapunov analysis of general GPAW compensated systems

Recent Results

- When restricted to 1st-order constrained LTI system driven by 1st-order LTI controller, can show
  - GPAW compensated system is a **projected dynamical system** (PDS) (Dupuis and Nagurney 1993, Zhang and Nagurney 1995, Nagurney and Zhang 1996, Cojocaru and Jonker 2004) (PDS is an independent line of research that has attracted significant attention of economists, physicists and mathematicians)
  - GPAW scheme can only **maintain/enlarge exact** region of attraction
- When output equation of nominal controller depends **only on controller state**, $u = g_c(x_c)$, ie. not on measurements and/or exogenous inputs, then **exact** state-output consistency achieved when appropriately initialized - **eliminates** previously identified limitations
- Under similar conditions, derived geometric bounding condition foreseen to aid in Lyapunov analysis of general GPAW compensated systems

Recent Results

- When restricted to 1st-order constrained LTI system driven by 1st-order LTI controller, can show
  - GPAW compensated system is a **projected dynamical system** (PDS) (Dupuis and Nagurney 1993, Zhang and Nagurney 1995, Nagurney and Zhang 1996, Cojocaru and Jonker 2004) (PDS is an independent line of research that has attracted significant attention of economists, physicists and mathematicians)
  - GPAW scheme can only **maintain/enlarge exact** region of attraction
- When output equation of nominal controller depends **only on controller state**, $u = g_c(x_c)$, ie. not on measurements and/or exogenous inputs, then **exact** state-output consistency achieved when appropriately initialized - **eliminates** previously identified limitations
- Under similar conditions, derived geometric bounding condition foreseen to aid in Lyapunov analysis of general GPAW compensated systems

References I

K. J. Åström and L. Rundqwist. Integrator windup and how to avoid it. In *Proc. American Control Conf.*, pages 1693 – 1698, Pittsburgh, PA, June 1989.

D. S. Bernstein and A. N. Michel. A chronological bibliography on saturating actuators. *Int. J. Robust Nonlinear Control*, 5(5):375 – 380, 1995. doi: 10.1002/rnc.4590050502.

J.-P. Calvet and Y. Arkun. Feedforward and feedback linearization of nonlinear systems and its implementation using internal model control (IMC). *Ind. Eng. Chem. Res.*, 27(10):1822 – 1831, Oct. 1988. doi: 10.1021/ie00082a015.

M.-G. Cojocaru and L. B. Jonker. Existence of solutions to projected differential equations in Hilbert spaces. *Proc. Am. Math. Soc.*, 132(1):183 – 193, Jan. 2004. doi: 10.1090/S0002-9939-03-07015-1.

P. Dupuis and A. Nagurney. Dynamical systems and variational inequalities. *Ann. Oper. Res.*, 44(1):7 – 42, Feb. 1993. doi: 10.1007/BF02073589.

C. Edwards and I. Postlethwaite. Anti-windup and bumpless-transfer schemes. *Automatica*, 34(2):199 – 210, Feb. 1998. doi: 10.1016/S0005-1098(97)00165-9.

G. Herrmann, M. C. Turner, P. Menon, D. G. Bates, and I. Postlethwaite. Anti-windup synthesis for nonlinear dynamic inversion controllers. In *Proc. 5th IFAC Symp. Robust Control Design*, volume 5, Toulouse, France, July 2006.

A. S. Hodel and C. E. Hall. Variable-structure PID control to prevent integrator windup. *IEEE Trans. Ind. Electron.*, 48(2):442 – 451, Apr. 2001. doi: 10.1109/41.915424.

Q. Hu and G. P. Rangaiah. Anti-windup schemes for uncertain nonlinear systems. *IET Control Theory Appl.*, 147 (3):321 – 329, May 2000. doi: 10.1049/ip-cta:20000136.

P. A. Ioannou and J. Sun. *Robust Adaptive Control*. Prentice Hall, Upper Saddle River, NJ, 1996. URL `http://www-rcf.usc.edu/~ioannou/Robust_Adaptive_Control.htm`.

E. N. Johnson and A. J. Calise. Limited authority adaptive flight control for reusable launch vehicles. *J. Guid. Control Dyn.*, 26(6):906 – 913, Nov. – Dec. 2003.

References II

N. E. Kahveci, P. A. Ioannou, and M. D. Mirmirani. A robust adaptive control design for gliders subject to actuator saturation nonlinearities. In *Proc. American Control Conf.*, pages 492 – 497, New York, NY, July 2007. doi: 10.1109/ACC.2007.4283128.

N. Kapoor and P. Daoutidis. Stabilization of systems with input constraints. *Int. J. Control*, 66(5):653 – 675, Mar. 1997. doi: 10.1080/002071797224496.

N. Kapoor and P. Daoutidis. An observer-based anti-windup scheme for non-linear systems with input constraints. *Int. J. Control*, 72(1):18 – 29, Jan. 1999. doi: 10.1080/002071799221361.

T. A. Kendi and F. J. Doyle. An anti-windup scheme for multivariable nonlinear systems. *J. Process Control*, 7(5): 329 – 343, Oct. 1997. doi: 10.1016/S0959-1524(97)00011-5.

H. K. Khalil. *Nonlinear Systems*. Prentice Hall, Upper Saddle River, NJ, 3 edition, 2002.

M. V. Kothare, P. J. Campo, M. Morari, and C. N. Nett. A unified framework for the study of anti-windup designs. *Automatica*, 30(12):1869 – 1883, Dec. 1994. doi: 10.1016/0005-1098(94)90048-5.

P. P. Menon, G. Herrmann, M. C. Turner, D. G. Bates, and I. Postlethwaite. General anti-windup synthesis for input constrained nonlinear systems controlled using nonlinear dynamic inversion. In *Proc. 45th IEEE Conf. Decision and Control*, pages 5435 – 5440, San Diego, CA, Dec. 2006. doi: 10.1109/CDC.2006.377291.

P. P. Menon, G. Herrmann, M. Turner, M. Lowenberg, D. Bates, and I. Postlethwaite. Nonlinear dynamic inversion based anti-windup - an aerospace application. In *Proc. 17th IFAC World Congress*, volume 17, pages 14156 – 14161, Seoul, Korea, July 2008a.

P. P. Menon, G. Herrmann, M. C. Turner, M. Lowenberg, D. Bates, and I. Postlethwaite. Dynamic wind tunnel rig implementation of nonlinear dynamic inversion based anti-windup scheme. In *Proc. AIAA Guidance Navigation and Control Conf. and Exhibit*, Honolulu, HI, Aug. 2008b. AIAA–2008–7166.

F. Morabito, A. R. Teel, and L. Zaccarian. Nonlinear antiwindup applied to Euler-Lagrange systems. *IEEE Trans. Robot. Autom.*, 20(3):526 – 537, June 2004. doi: 10.1109/TRA.2004.824933.

A. Nagurney and D. Zhang. *Projected Dynamical Systems and Variational Inequalities with Applications*. Int. Ser. Oper. Res. Manag. Sci. Kluwer, Norwell, MA, 1996.

References III

J.-B. Pomet and L. Praly. Adaptive nonlinear regulation: Estimation from the Lyapunov equation. *IEEE Trans. Autom. Control*, 37(6):729 – 740, June 1992. doi: 10.1109/9.256328.

J. B. Rosen. The gradient projection method for nonlinear programming. part I. linear constraints. *J. Soc. Ind. Appl. Math.*, 8(1):181 – 217, Mar. 1960.

J. B. Rosen. The gradient projection method for nonlinear programming. part II. nonlinear constraints. *J. Soc. Ind. Appl. Math.*, 9(4):514 – 532, Dec. 1961.

J.-J. E. Slotine and J. A. Coetsee. Adaptive sliding controller synthesis for non-linear systems. *Int. J. Control*, 43 (6):1631 – 1651, June 1986. doi: 10.1080/00207178608933564.

S. Tarbouriech and M. Turner. Anti-windup design: an overview of some recent advances and open problems. *IET Control Theory Appl.*, 3(1):1 – 19, Jan. 2009. doi: 10.1049/iet-cta:20070435.

S.-S. Yoon, J.-K. Park, and T.-W. Yoon. Dynamnic anti-windup scheme for feedback linearizable nonlinear control systems with saturating inputs. *Automatica*, 44(12):3176 – 3180, Dec. 2008. doi: 10.1016/j.automatica.2008.10.003.

D. Zhang and A. Nagurney. On the stability of projected dynamical systems. *J. Optim. Theory Appl.*, 85(1):97 – 124, Apr. 1995. doi: 10.1007/BF02192301.