Onboard GPS Signal Augmentation for Spacecraft Formation Flying

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BIOGRAPHY

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ABSTRACT

Formation flying of multiple vehicles requires accurate information on the relative position and attitude between the vehicles. Carrier Phase Differential GPS (CDGPS) can provide the necessary accuracy in position and attitude sensing to successfully carry out such missions. In missions with little to no visibility to the NAVSTAR constellation, CDGPS can still be used by augmenting the available signals with onboard transmitters broadcasting the L_1 carrier signal modulated by a unique C/A code. The feasibility of this augmented GPS system is demonstrated with a two-vehicle formation in an indoor environment.

1 INTRODUCTION

Many future space missions involving formation flying among multiple spacecraft have been proposed, including DS-3, EO-1, GRACE, and MUSIC [1, 2, 3]. One of the driving forces behind the expansion to multiple spacecraft is a desire for larger size spacecraft. By separating a single structure into smaller pieces that collectively *act* as a single structure, virtual spacecraft of unlimited size can be flown. This will enable new scientific missions, based on distributed yet coordinated measurements, such as stellar interferometry and gravity field mapping.

The concept of coordinating multiple vehicles to act as a single structure is the idea behind formation flying. To ensure the success of this behavior, one of the fundamental requirements is the ability to accurately sense relative position and attitude between the vehicles. CDGPS has been shown to be an accurate method of determining relative position and attitude [4]. Yet ordinarily, using CDGPS would limit a mission to an orbit in which adequate visibility to the NAVSTAR constellation is ensured, typically LEO [5]. For higher elevation missions, such as in a GEO orbit, visibility can easily fall to two or even zero satellites [6, 7]. With only two satellites visible, insufficient measurements are available for a formation of vehicles to solve for relative position and attitude between all vehicles. Before CDGPS can be useful beyond LEO, the signals from the visible NAVSTAR satellites must be augmented by additional GPS signals.

Signal generators transmitting GPS-like signals have been shown to provide a GPS receiver with information equivalent to that from a real satellite [8, 9]. These signal generators are called pseudolites. A logical way to augment the signals available to a spacecraft formation is to place pseudolites *as well as* receivers onboard the vehicles themselves [10]. Enough signals are then available for the formation to solve for position and attitude, even if no NAVSTAR satellites are visible. Thus, this technique could also be used for relative positioning in deep space, where the NAVSTAR satellites are not accessible. This research examines the feasibility of onboard signal augmentation using GPS transmitters. First, a theoretical examination of the ability to solve for relative positions and attitudes is presented, followed by an experimental validation of the ideas.

2 PROBLEM STATEMENT

The case studied in this paper is a formation of three vehicles, in a high orbit such as GEO or in deep space. This work is also applicable to a formation of vehicles on the ground with limited visibility to the satellites, as well as applications requiring increased precision and integrity attained by additional measurements. The vehicles are assumed to be separated by baselines on the order of 0.5-1 km. These baselines were chosen to be of similar magnitude to those proposed for NASA's New Millennium Interferometer [11].

The pseudolite signal structure is assumed to be an L_1 carrier (frequency of approximately 1575.42 MHz) modulated by a C/A code. Previous simulation studies have been done [10] in which a much higher frequency signal is used in order to improve position and attitude accuracy. However, the emphasis of this work is to develop a system compatible with standard GPS technology and to take advantage of any available measurements from the NAVSTAR satellites.

It is assumed that one vehicle will be outfitted with an absolute attitude sensor, and will have knowledge of its position or orbital elements as well as a source of timing information. This assumption is only necessary if NAVSTAR satellites are to be incorporated in the solution. For an application in which no satellites are visible, the augmented GPS system does not require absolute position and attitude information. Yet, these are quantities that usually need to be known, regardless of the GPS positioning system. Therefore, it is reasonable to assume this information is available.

The coordinate system developed for this problem is shown in Figure 1. One vehicle is defined as the lead vehicle, and the origin of the coordinate system is placed at its center of mass. The x axis is defined to coincide with the center of mass of the second vehicle. The $x-y$ plane is the plane containing all three vehicles. The z axis is orthogonal to the $x-y$ plane. This frame will be referred to as the locally level frame. The unknowns in this frame are then the ranges to vehicles two and three, and three rotation matrices defining the rotation from each vehicle's local frame to the locally level frame. The vector from the lead vehicle to the second vehicle, χ_1, can be defined by a single coordinate, x_2. The vector from the first vehicle to the third vehicle, χ_3 can be defined by two coordinates, x_3 and y_3.

Since the lead vehicle is assumed to have an absolute position sensor onboard, the lines-of-sight (LOS) to any available NAVSTAR satellites are assumed to be known in an earth-centered, earth-fixed frame. The LOS for any particular satellite can be assumed to be the same for all vehicles in the formation since the vehicles are separated by a small distance compared to the distance to the NAVSTAR satellites.

A receiver with four antenna inputs such as the TANS

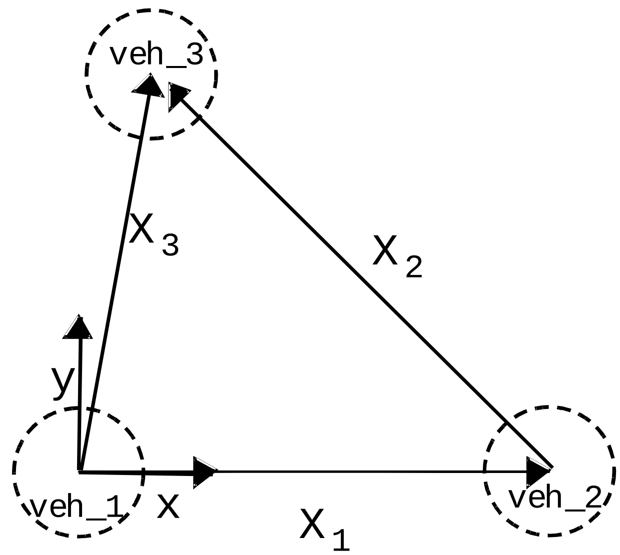

Fig. 1: Coordinate System Definition

Quadrex is considered, in which three independent receive antenna baselines exist per vehicle.

Two types of measurements are of interest. The first type of measurement is a difference between antennas on the same vehicle, or an *intra-vehicle* difference. These measurements yield information on the attitude of the vehicle. The second type of measurement is an *inter-vehicle* measurement taken between antennas on separate vehicles. These measurements yield information on the relative position between vehicles. For the vehicle separations considered, many but not all of these GPS measurement equations can be linearized by assuming planar wavefronts. The magnitude of position error expected with a planar wavefront assumption is $\approx b^2/2d$, where b is the baseline between two antennas and d is the distance to the transmitter[12]. Therefore, measurements formed with the NAVSTAR satellites can be linearized, since d is the distance to the satellite and b is at most the distance between two vehicles (for inter-vehicle differences). For equations formed with the onboard transmitters, the assumption of linearity can only be made for intra-vehicle differences. For inter-vehicle differences, the distance to the transmitter is of similar magnitude as the distance between antennas.

2.1 CODE POSITIONING

Differential code phase positioning can be employed for an initial estimate of the range between all vehicles in the formation. By taking single differences between master antennas on two separate vehicles, the code phase equations can be written as:

$$\begin{bmatrix} \Delta\tau_{12}^1 \\ \Delta\tau_{13}^1 \\ \Delta\tau_{12}^2 \\ \Delta\tau_{23}^2 \\ \Delta\tau_{13}^3 \\ \Delta\tau_{23}^3 \end{bmatrix} \approx \begin{bmatrix} |B_1 - Q_1| - |\chi_1| + (b_1 - b_2) \\ |B_1 - Q_1| - |\chi_3| + (b_1 - b_3) \\ |\chi_1| - |B_2 - Q_2| + (b_1 - b_2) \\ |B_2 - Q_2| - |\chi_2| + (b_1 - b_3) - (b_1 - b_2) \\ |\chi_3| - |B_3 - Q_3| + (b_1 - b_3) \\ |\chi_2| - |B_3 - Q_3| + (b_1 - b_3) - (b_1 - b_2) \end{bmatrix} \quad (1)$$

where τ_{jk}^i is the single differenced pseudorange in units of distance between vehicles j and k, from vehicle i, χ is the true range vector as defined in Figure 1, and b is the receiver clock error indexed by vehicle. Equation 1 includes measurements of the transmitter on a particular vehicle transmitting to the receiver on the same vehicle. The availability of this measurement will be shown in Section 3. The line biases are grouped into the clock biases, and these clock biases are written relative to the clock error of the receiver on vehicle 1. Therefore, there are five unknowns in these equations, namely the three ranges, and the two relative clock errors of the receivers. The transmitter clock errors cancel out in the single differencing.

Equation 1 assumes the distance between the receive antenna on one vehicle, and the transmit antenna on another vehicle is equal to the magnitude of the vector between the centers of mass of the vehicles. If approximations of the vehicle attitudes are known, the vectors from the centers of mass of the vehicles to the receive and transmit antennas can be expressed in the local frame. These vectors can then be included in the code equations. The error introduced by omitting these vectors is on the order of their magnitude. For antennas located 1 meter from the center of mass, this error is well within the noise of the code pseudorange. Once the GPS solution to the attitude is calculated, that information can be incorporated to improve the accuracy of the solution.

The code measurement equations are formed with measurements from the onboard transmitters only. Even if NAVSTAR satellites are visible, the information cannot initially be incorporated in the code solution. The lines-of-sight to the satellites (unit vectors in the direction of the satellites) must be expressed in the local coordinate frame, which requires knowledge of the attitude of the lead vehicle relative to the local frame. Therefore, until the attitude is found, all solutions are carried out with onboard transmitters only.

The dilution of precision (DOP) for the states as formed in Equation 1 were examined for inter-vehicle baselines ranging from 500 m to 1 km. The states under consideration are the position coordinates comprising the range vectors χ_1, χ_2, and χ_3. For the coordinate system as described in Figure 1, these ranges can be described by a single coordinate, x_2 to vehicle 2, and two coordinates, x_3 and y_3 to vehicle 3. Combinations of inter-vehicle distances in the range specified were examined, excluding the case of all three vehicles in a line (*e.g.* if $|\chi_1| = 1000, |\chi_2| = 500, |\chi_3| = 500$). The mean values of the DOPs for these three coordinates over this range of baselines are 0.71, 1.15, and 0.74 respectively. These DOPs suggest a positioning error on the order of the standard deviation of the noise on the code. A reasonable estimate of the noise for the onboard transmitters is 3 meters [13]. Therefore, range accuracies are expected to be around 3 meters.

Equation 1 is written in terms of single differences since there are not enough unique double differences with just the onboard transmitters to solve for all three range unknowns. However, after the attitude is known, the equations could be written in terms of double differences if satellites are incorporated in the solution. The transmitters used for the differences would be a combination of onboard transmitters as well as NAVSTAR satellites. The advantage of forming double differences is that the receiver clock biases cancel out in the differencing.

2.2 ATTITUDE

The attitude of each vehicle must be found relative to the locally level frame. Using carrier phase GPS measurements from the onboard transmitters and four antennas on each vehicle, the attitude of each vehicle can be found independently. The measurements of interest are single differences between pairs of antennas on the vehicle. Consider solving for the attitude of vehicle 1. Carrier phase measurements can be taken from the two transmitters on vehicles 2 and 3 to each of the three pairs of antennas, or baselines, on vehicle 1. This produces six equations. Expressing the unknowns in terms of a rotation matrix from the vehicle frame to the locally level frame, the equations can be written as:

$$[\Delta\phi] + \lambda N + \beta = \begin{bmatrix} LOS_{v2} \cdot (R \cdot BL_1) \\ LOS_{v2} \cdot (R \cdot BL_2) \\ LOS_{v2} \cdot (R \cdot BL_3) \\ LOS_{v3} \cdot (R \cdot BL_1) \\ LOS_{v3} \cdot (R \cdot BL_2) \\ LOS_{v3} \cdot (R \cdot BL_3) \end{bmatrix} \quad (2)$$

where β is the line bias vector, R is the rotation matrix, the components of which are the states to be evaluated, N is the integer ambiguity vector, LOS_{v2} is the line-of-sight to vehicle 2 in the locally level frame, and BL_i is the baseline vector from the master antenna to antenna i in the vehicle frame. All antennas are hooked to a common receiver clock, so no clock errors remain in these equations. The lines-of-sight to the vehicles can be described in the locally level frame in terms of the known ranges from the code ranging solution. These three ranges, $|\chi_1|, |\chi_2|$, and $|\chi_3|$, fully determine the distances and angles of the triangle formed by the three vehicles. Given that the distances between the vehicles are large in comparison to the baselines of the antennas, this problem becomes a standard attitude problem as often seen in the literature [14]. Many methods of solving for the integer ambiguity exist [15, 16]. Once the ambiguity is resolved, an estimator such as an extended Kalman filter can be used to resolve the attitude.

Attitude DOPs for a variety of baselines and geometries were examined in order to estimate the predicted accuracy of the attitude solution. Consider a set of four antennas on one vehicle, pointing directly towards two collocated transmitters. Now, varying the position of the transmitters until they are at either side of the antenna array, the angle between the transmitters will have spanned 0° to 180°. With this definition of the angle between the transmitters, Figure 2 shows the DOPs of the Euler angle states defining the rotation of the vehicle vs. the angle between the transmitters. The baselines considered are similar to those used in the experimental work described in Section 3: approximately 0.7 meters. For the range of separations between 30° and 150°, the mean DOPs of the three states are: 1.88, 2.2, and 1.4 for pitch, roll, and yaw respectively. With noise values on the carrier phase single differences of $\sigma = 0.5$ cm, this results in attitude errors of 0.9, 1.1, and 0.7 degrees for pitch, roll, and yaw. These values will vary depending on baselines and geometry. As the baselines between antennas are increased, accuracy improves.

Once the attitude of each vehicle is known, any available NAVSTAR satellite measurements can be incorporated. These extra measurements are particularly useful for the carrier phase positioning solutions, discussed in the following section, and for improving the accuracy of the attitude and code solutions. From knowledge of the lead vehicle's orbital elements, a measure of time, and knowledge of the orbital elements of the NAVSTAR satellites, the LOS to the satellites are known in an ECEF frame. Additionally, the rotation matrix from the ECEF frame to the vehicle frame is known from an attitude sensor such as a star tracker on the lead vehicle (or from a GPS solution if three satellites are visible). From the computed GPS attitude solution, the rotation matrix from the vehicle frame to the locally level frame is now also known.

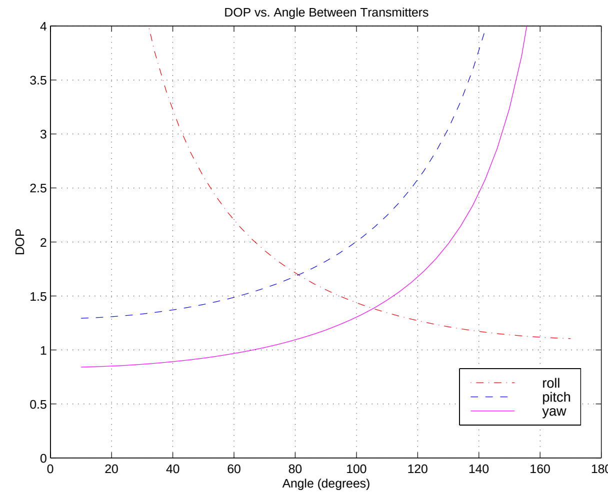

Fig. 2: Dilution of Precision for Attitude States vs. Separation Between Transmitters

Therefore, the LOS to the NAVSTAR satellites in a locally level frame can be found as:

$$LOS_{LL}^{sat} = R_{LL}^{veh} R_{veh}^{ECEF} LOS_{ECEF}^{sat} \quad (3)$$

where LOS_{LL}^{sat} is the line-of-sight to a particular satellite in the locally level frame, LOS_{ECEF}^{sat} is the line-of-sight to a NAVSTAR satellite in an ECEF frame, R_{LL}^{veh} is the rotation matrix from the vehicle frame to the locally level frame, and R_{veh}^{ECEF} is the rotation matrix from the ECEF frame to the vehicle frame. With the calculations given in Equation 2, and assumptions stated in the beginning, each of the matrices on the right hand side of Equation 3 are now known.

2.3 CARRIER PHASE POSITIONING

In order to calculate precise estimates (within several centimeters) of the relative positions of the vehicles, differential carrier phase positioning techniques must be employed. The equations will vary depending on whether or not satellites are visible, and how many are visible. The equations will all include an integer ambiguity that must be resolved.

If satellites are visible, the measurement equations can be formed as double differences between master antennas on separate vehicles and two transmitters. This is exactly analogous to the code range solution, when satellites are available. All clock biases can be subtracted out and the relative positions between vehicles are observable. The equations in this case will include a combination of satellite transmitters, and onboard transmitters. Assuming visibility to two satellites, these equations can be written as:

$$\begin{bmatrix} \nabla\Delta\phi^{s_1 s_2}_{v_1 v_2} \\ \nabla\Delta\phi^{s_1 s_2}_{v_2 v_3} \\ \nabla\Delta\phi^{v_2 v_1}_{v_2 v_3} \\ \nabla\Delta\phi^{v_2 v_3}_{v_3 v_1} \\ \nabla\Delta\phi^{v_3 v_1}_{v_1 v_2} \end{bmatrix} + \lambda \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{bmatrix} = \begin{bmatrix} LOS_{s_1} \cdot b_1^2 - LOS_{s_2} \cdot b_1^2 \\ LOS_{s_1} \cdot b_2^3 - LOS_{s_2} \cdot b_2^3 \\ h_1(X) \\ h_2(X) \\ h_3(X) \end{bmatrix} \quad (4)$$

where the superscripts on $\nabla\Delta\phi$ represent the two transmitting vehicles and the subscripts represent the two receiving vehicles, s_k represents the kth satellite, v_k is the kth vehicle, b^i_j is the baseline vector from the master antenna on vehicle i to the master antenna on vehicle j in the local frame, the LOS are expressed in the local frame, and $h(X)$ is a nonlinear equation of the state, containing both the relative attitudes and positions of the vehicles. As an example, $h_1(X)$ is written below.

$$h_1 = \left[|(B_2 - Q_2)| - \left|(\chi_3 + R^{v3}_{LL} B_3) - (\chi_1 + R^{v2}_{LL} Q_2)\right| \right]$$
$$- \left[\left|\chi_1 + R^{v2}_{LL} B_2 - R^{v1}_{LL} Q_1\right| - \left|\chi_3 + R^{v3}_{LL} B_3 - R^{v1}_{LL} Q_1\right| \right]$$

χ_i is the distance vector defined in Figure 1, R^{vi}_{LL} is a rotation matrix from vehicle i's frame, to the locally level frame, Q_i is the transmitter baseline on vehicle i, and B_i is the master antenna baseline on vehicle i.

One method of accurately estimating the integer parameters in Equation 4 involves collecting data over time, while the vehicles undergo large relative motion [12]. Equation 4 is stacked in time, to create a large matrix, which is solved in a batch solution process. While this method is accurate, it is dependent on the large relative motion.

If no satellites are visible, the motion technique can still be applied to resolve integers. But, in this case, the data should be collected when the vehicles are in close proximity, and still in a nonlinear regime. With the smaller separations, the multi-epoch data matrix is better conditioned, and more unique measurements are available due to the coupling between attitude and position. The equations are similar to Equation 4, except in place of the linear double differences with satellites, single differences of antennas on the same vehicle are used. The resultant equation for a single epoch of data is written in Equation 5.

$$\begin{bmatrix} \Delta\phi^1_{2j} \\ \Delta\phi^1_{3j} \\ \Delta\phi^2_{3j} \\ \Delta\phi^3_{2j} \\ \nabla\Delta\phi^{v_2 v_1}_{v_2 v_3} \\ \nabla\Delta\phi^{v_2 v_3}_{v_3 v_1} \\ \nabla\Delta\phi^{v_3 v_1}_{v_1 v_2} \end{bmatrix} = \begin{bmatrix} h(X_1, X_2) \\ h(X_1, X_3) \\ h(X_2, X_3) \\ h(X_2, X_3) \\ h(X_1, X_2, X_3) \\ h(X_1, X_2, X_3) \\ h_(X_1, X_2, X_3) \end{bmatrix} + \lambda N \quad (5)$$

where $\Delta\phi^i_{kj}$ represents the single difference phase measurements from the transmitter on vehicle i to all antenna baselines indexed by j on vehicle k, $h(X_i, X_j)$ is a set of nonlinear equations which are functions of the attitudes of vehicles i and j and the relative distances between vehicles i and j, and the other parameters are the same as defined in Equation 4.

This technique shows the feasibility of integer resolution, but is not necessarily the optimal method of resolution. Large motion can be very expensive for spacecraft. Yet, if the motion is carried out when the vehicles are first released from a launch vehicle, and thus in near proximity, this expense may be acceptable. Other more optimal techniques are under investigation, including the use of multiple frequencies for integer resolution[17, 18].

After estimating the initial set of integers, the motion technique is not necessary for future signal outages and reacquisitions. Each measurement has associated with it a particular integer that remains constant as long as that measurement is in lock. But if the measurement is lost, then regained, the integer must be recalculated. Conditional on receiving enough signals from the augmented GPS system to resolve the position states, the integer for a newly acquired signal can be computed directly, without undergoing any motion. A simplistic method of recomputing this integer is to augment the state vector with the unknown integer, and keep the integers associated with all of the old measurements constant. For example, assuming the last measurement has just been acquired, the integer for that measurement could be computed as:

$$\left[\nabla\Delta\phi \right] + \lambda \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} & 0 \\ & 0 \\ H & 0 \\ & 0 \\ & -1 \end{array} \right] \begin{bmatrix} x \\ N_5 \end{bmatrix} \quad (6)$$

Once the integer ambiguity has been resolved, carrier phase positioning can be used to provide position estimates within several centimeters. When satellites are visible, the equations can continue to be written as in Equation 4 with the known value of the integers substituted in the integer vector. If no satellites are visible, as the baselines between the vehicles increase beyond the nonlinear regime, Equation 5 is no longer applicable. Instead, the measurement equations can be written in terms of inter-vehicle single differences among just the vehicles themselves. The carrier phase equations will then be completely analogous to the code ranging equations written in Equation 1, which were also formed assuming no availability of satellites.

3 EXPERIMENTAL WORK

As shown in Section 2, placing GPS transmitters onboard three vehicles will provide enough measurements for a relative position and attitude solution for all vehicles in the formation, even in the absence of visibility to NAVSTAR satellites. For this technique to be implemented using conventional receivers and pseudolites, it is necessary to receive signals over a wide range of distances: from less than 1 meter, to the maximum distance between vehicles in the formation. The emphasis of this research is to demonstrate relative position and attitude computation on a formation of three vehicles, using onboard transmitters with standard GPS hardware.

3.1 TESTBED

To study formation flying issues, a formation flying testbed has been created in the Stanford Aerospace Robotics Laboratory. It consists of three active free-flying vehicles that move on a 12 ft $\times$ 9ft granite table top. These air cushion vehicles simulate the zero-g dynamics of a spacecraft formation in a plane. The vehicles are propelled by compressed air thrusters. Power is provided to the vehicles by batteries, and all computation is done by the vehicles using Motorola PowerPC real-time processors. The vehicles communicate with each other over a wireless ethernet.

An indoor GPS environment was originally constructed for the experiments by Zimmerman[4]. This consists of six ceiling mounted pseudolite transmitters broadcasting the L_1 carrier signal modulated by a unique C/A code, with no navigation data. One transmitter is designated the *master pseudolite* and transmits a 50 bps data message modulated on top of the C/A code, which contains timing information for synchronizing the carrier phase measurements of multiple receivers. This synchronization enables differential carrier phase measurements to be made between receivers on separate vehicles.

The indoor environment has been extended to include a transmitter onboard each vehicle. The GPS signal is produced by an IN200c signal generator, fabricated by Integrinautics. Again, each signal is an L_1 carrier phase signal, with a unique C/A code. The signal is transmitted through a dipole antenna on top of each vehicle (see Figures 3 and 4). Less multipath rejection is possible with the linear signal sent by a dipole antenna, yet it was chosen based on a need for an omni-directional transmitter in the 2-D workspace. The transmitted signal must be visible to the other vehicles, independent of the orientation of the transmitting vehicle. This will be particularly important when the formation is being initialized, and the vehicle attitudes are initially unknown.

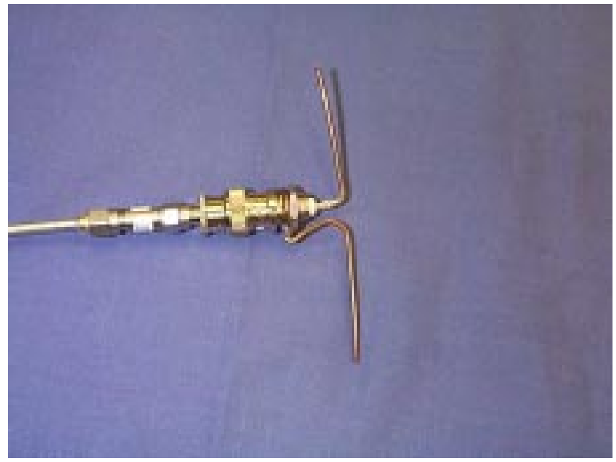

Fig. 3: Dipole Antenna for Signal Transmission

For receiving the signal, each vehicle has a six channel Trimble TANS Quadrex receiver with customized software. Four receive antennas are mounted on each vehicle.

Finally, for a truth comparison, a vision system is mounted above the table which tracks a unique LED pattern on each vehicle. The vision system has an absolute accuracy better than 1 cm for position and 0.5° for attitude throughout the workspace.

3.2 TEST DESCRIPTION

Initial testing was carried out using two vehicles. The GPS signals were provided by the two onboard transmitters, as well as two ceiling mounted pseudolites. This setup is analogous to a spacecraft formation which only has visibility to two NAVSTAR satellites. Without additional signals, the relative position and attitude between the vehicles would be unsolvable. By placing transmitters onboard the vehicles themselves, enough measurements become available to solve for the states of interest (position and attitude). An important distinction between this test and a mission in space is the small separation between the vehicles. In the indoor setting, the vehicles are separated by several meters, therefore a planar wavefront assumption cannot be made. This presents a nonlinear set of equations in which the attitude and the positions between the vehicles are coupled. Therefore, the attitude problem cannot be separated from the positioning problem as was done in Section 2.

For all of the data presented here, integer initialization is done with the vision system at startup, and all subsequent integer calculations resulting from signal

Fig. 4: Vehicle with Transmitter and Receiver

acquisitions are done from the GPS solution.

3.3 EXPERIMENTAL RESULTS

Before a solution for position and attitude is presented, the signal quality of the transmitted signal is examined. Of particular importance is the performance of the receiver when a transmitter is located less than one meter away. The most basic need is for that receiver to be able to lock on to the available signals. It was discovered that a continuously transmitting pseudolite would jam the receiver that was situated on the same vehicle. The receiver would report SNRs of less than 5 on all six channels, suggesting no valid signals were in lock. A solution to this problem is suggested by Elrod and Van Dierendonck [13]. By pulsing the transmitter so that it only broadcasts for a fraction of each millisecond, the receiver is able to lock on not only to the signals of other transmitters in the room, but also the transmitted signal from the same vehicle. The latter affect is particularly important for simplifying the transmitter and receiver design. The ability to receive the local signal eliminates the need for tying together the clocks on the transmitter and receiver.

By adjusting the pulsewidth of the transmitted signal, the receiver is able to not only pick up the pulsed signals, but continuously maintain lock on these channels. For the TANS Quadrex receiver, a duty cycle of about 14% is sufficient to track the signal continuously. The duty cycle is slightly higher than would be necessary in a different receiver, since the TANS multiplexes between four antennas. In fact, a receiver that does not multiplex would most likely be preferable for receiving a pulsed signal, and would reduce the noise levels. The TANS is already only listening to a particular antenna for 25% of the time. A pulsed signal reduces the amount of signal seen by each antenna even further. However, the choice was made to continue using the TANS Quadrex for initial testing since it was available, and all software to receive and synchronize the pseudolite signals was already written.

The signal to noise levels previously seen in this environment using continuously transmitting pseudolites with helical antennas were typically in the range of 10-20, depending on location in the workspace. These pseudolites were several meters away from the receivers. Figure 5 depicts the signal to noise ratio (SNR) of the signal transmitted by a dipole antenna on vehicle 2 and received by vehicle 1. The scale is plotted with a minimum value of five, since below this level, the signal is most likely not in lock. Despite the vehicles remaining stationary over the time of data collection, the SNR changes rapidly over a range of about 6 (Note that for the TANS Quadrex, the SNR output is a linear scale, not in dB. The value refers to the C/A code correlation value of the received signal). This fluctuation had been seen in previous work when the vehicles were moving, thus experiencing rapid changes in the multipath environment. In this case, the fluctuations are due to the noisy environment, as well as the use of a pulsed signal. The pulsing is done in a pseudorandom pattern, and therefore at different times the master antenna will receive differing amounts of the signal. Additionally, due to the omni-directional nature of the dipole transmit antennas, the multipath in the indoor environment is increased. In spite of the fluctuations, the signal remains in constant lock, and therefore is a usable signal in forming measurements. Figure 6 shows the SNR of the signal transmitted by vehicle 1 and received by vehicle 1. Again, the signal remains in constant lock, in spite of slight variations. The actual mean values of the SNRs of both pulsed signals from the onboard transmitters have been chosen at these levels shown in Figures 5 and 6 in order to keep the signal strength consistent with the signals received from the ceiling mounted pseudolites, and roughly equivalent for all channels. The value itself is adjusted through attenuation added between the signal generator and the transmit antenna.

Earlier work with this testbed using only ceiling

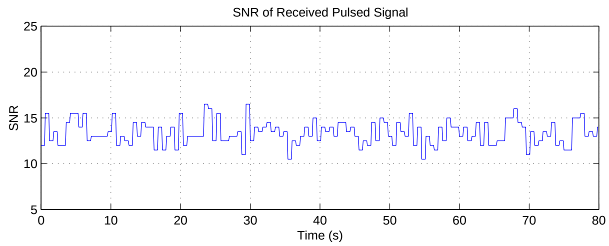

Fig. 5: SNR of a Pulsed Signal Received from Several Meters

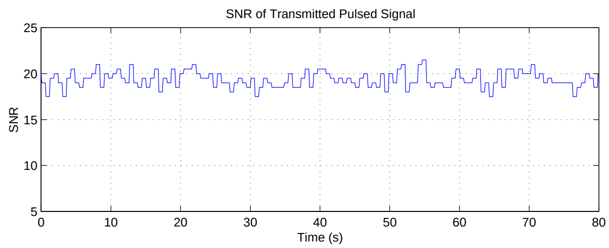

Fig. 6: SNR of a Pulsed Signal Received from Less than 0.5 Meters

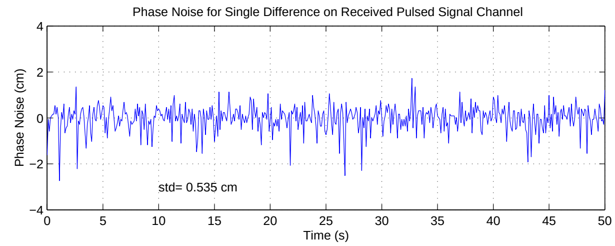

Fig. 7: Phase Noise on a Single Difference Measurement

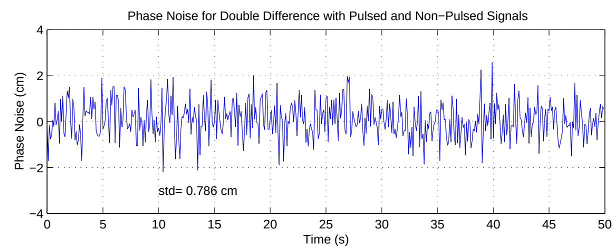

Fig. 8: Phase Noise on a Double Difference Measurement

mounted pseudolites with helical antennas typically yielded a 0.25 cm standard deviation in phase noise. With the onboard dipole transmitters using pulsed signals, the noise level was expected to be higher. The noise level on the carrier phase measurement between two antennas on a single vehicle is plotted in Figure 7. The standard deviation of the noise has increased to 0.5 cm. This agrees with expectations: since the signal is only broadcast a fraction of the time, the noise level is expected to be larger than an equivalent continuously broadcast signal.

Figure 8 shows the noise level on a carrier phase measurement taken between master antennas on two different vehicles, or an inter-vehicle measurement. This measurement is a double difference taken with an onboard transmitter and a ceiling mounted pseudolite as the two transmitters. The standard deviation of the noise in this case has increased to 0.8 cm which again follows trends seen in previous work. Double difference measurements between two different vehicles are typically noisier than single difference measurements on one vehicle. Even with this increase in noise, the noise levels are low enough to indicate that using the pulsed transmitters onboard the vehicles is feasible.

The first test is a stationary test, in which both vehicles are stationary on the table. One vehicle collects all phase measurements using the wireless ethernet to communicate with the other vehicle, and computes the relative position and attitude to the second vehicle. Truth data is collected from the vision sensing system for comparison. The resulting position errors in both x and y are plotted in Figure 9, for 50 seconds of data. The mean radial error $i.e.\sqrt{x^2+y^2}$ is 0.786 cm with $\sigma = 0.399$ cm. The distributions of the errors are shown in Figure 10. The relative position error is well contained within 2 cm for both coordinates.

The relative attitude states are also computed, and compared with the vision system solution. The resulting error in yaw is shown in Figure 11. While the attitude solution appears to be better than 0.5°, the vision system accuracy is only known to be at least 0.5°. Therefore this result may be within the noise of the truth sensor.

While static tests show the potential performance of the system, dynamic testing is equally important. Due to the large amount of multipath in the environment, small movements lead to significant changes in signal reception. Therefore, only small motions are considered. In this case, one vehicle remains grounded and the other vehicle moves randomly around in a 70 cm by 50 cm area. Even this small amount of motion is enough to alter the SNRs of the received signals. The errors in relative position in x and y coordinates are plotted in Figure 12 for 60 seconds of data. As expected, the solution has degraded slightly from the static tests, yet remains mostly within 2 cm. The mean of the radial error is 0.62 cm with $\sigma = 0.66$ cm. The few data points that deviate beyond a 2 cm error correspond to the vehicle losing lock on a signal for several seconds. The distributions of these position errors are plotted in Figure 13. For the y coordinate, 100% of the measurements are within a ± 2cm error, and for the x coordinate, over 90% of the measurements are within ± 2 cm. The difference in accuracy in x and y is simply a result of the geometry: where the vehicles

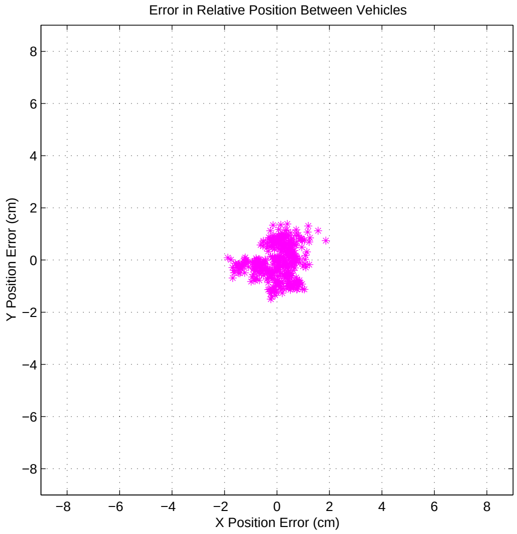

Fig. 9: Relative Position Errors in X and Y

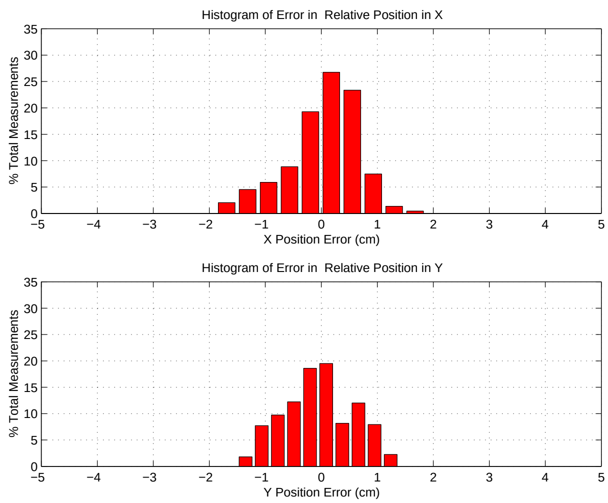

Fig. 10: Distribution of Relative Position Error in X (top) and Y (bottom)

are located and where the two stationary transmitters are located.

4 THREE VEHICLE SIMULATION

The experimental results presented in the previous section demonstrate the ability to transmit and receive GPS signals onboard the same vehicle, and to use these signals to accurately solve for position and attitude. In order to fully demonstrate the usefulness of this technique on a spacecraft formation, the experimental work must be extended to three vehicles. With three vehicles, the system can be completely independent of the ceiling mounted pseudolites. Simulations were run to determine the expected accuracy of a three vehicle formation. The cases of three vehicles with two stationary transmitters as well as three vehicles alone were examined. The noise value inputs to the sim-

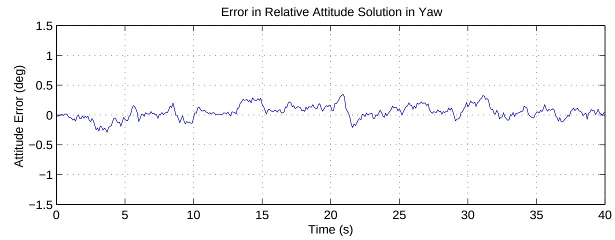

Fig. 11: Relative Attitude Error, in Yaw

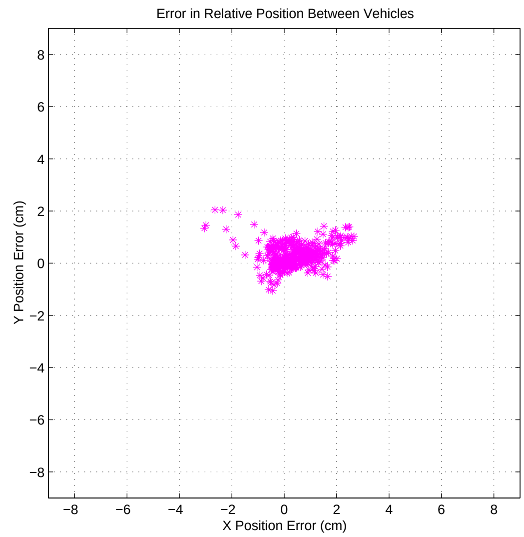

Fig. 12: Relative Position Errors in X and Y

ulation are derived from measurements taken on the vehicles themselves. For single differences, the noise is distributed as $N(0,[0.5\text{ cm}]^2)$ and for the double differences, the noise is distributed as $N(0,[1\text{ cm}]^2)$. With these noise values, the resultant accuracy in relative position is still within 2 cm for both cases. When no satellites (stationary pseudolites) are visible, the solution has slightly more error, but still remains within 2 cm. The attitude solution in yaw shows an error less than 0.5° for both cases, with the case of no satellites being slightly noisier. Therefore, the expansion to three vehicles is expected to yield similar accuracies as seen in the previous section.

The current formation flying testbed is being expanded for the three-vehicle tests. The vehicles will now reside on individual granite tables separated by several meters. This will allow larger baselines between the vehicles than are possible with the current testbed.

5 CONCLUSIONS

This research successfully demonstrates the use of onboard transmitters to augment available GPS signals in order to provide enough measurements to accurately solve for relative position and orientation of a two vehicle formation. This work is easily extendible to larger

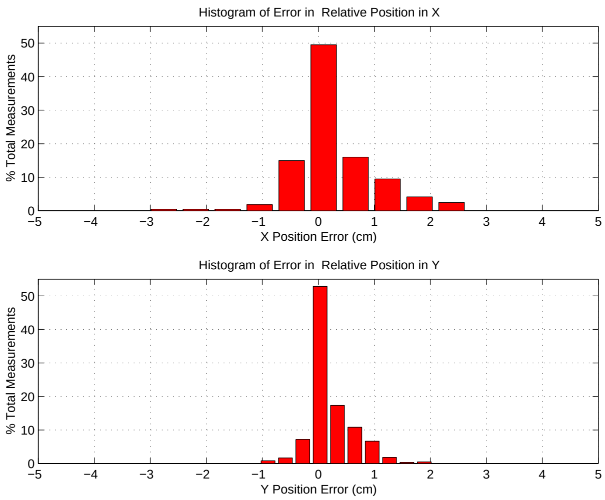

Fig. 13: Distribution of Relative Position Error in X (top) and Y (bottom)

formations, with present attention on a three-vehicle formation. Synchronization is not necessary between transmitters for the current testbed. The transmitters pulse for a set duration, at a random point in each code epoch. If the formation increases to much larger baselines between vehicles, it may be necessary to synchronize the transmitters due to a more significant near-far problem. The results presented, taken in an indoor environment, show accuracies of approximately 2 cm in position, and better than a degree in orientation. The noise environment inside the lab is significantly worse than would be expected in space. Therefore, these accuracies represent a conservative estimate of the capability of the augmented GPS system for space applications.

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