UAV TRAJECTORY DESIGN USING TOTAL FIELD COLLISION AVOIDANCE

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ABSTRACT

A system of N navigating vehicles, each associated with a start position and a goal position, may involve a large number of potential conflicts that grows exponentially with N and calls for a distributed rather than centralized control approach. This paper investigates the feasibility of a total field sensing approach for collision avoidance as an alternative to previously published techniques which assume perfect information sharing between vehicles.

INTRODUCTION

The problem of coordinating the navigation of a large number of vehicles so that they all reach their goal positions without colliding has attracted a great deal of attention lately. As the number N of vehicles grows, the problem rapidly becomes intractable with centralized control approaches. Local approaches have been proposed based on nearest-neighbor interactions and/or potential fields. However, many of these approaches assume that each vehicle knows which are its nearest neighbors and what are their positions. In fact, some techniques require that each vehicle knows the positions and velocities of all the other vehicles for such approaches to work.

Early local approaches include Reynolds work on flocking of so-called boids with simple local rules based on the positions and velocities of the vehicle's nearest neighbors.[11] A similar fixed-velocity version was proposed by Vicsek[13] in 1995.

The promising simulation results of such heuristic methods soon inspired other researchers to approach the problem analytically with the intention of proving the emergence of flocking as the result of applying simple local rules. The dominant approach has used potential functions and is represented by McInnes,[10] Leonard[8] and Passino.[5] The flocking is here obtained as the balance of a far-field attraction and a near-field repulsion between vehicles which gives a simple expression for the resulting collective dynamics - Lyapunov stability theory can then be used to prove asymptotic stability. While theoretically elegant, these approaches make the assumption that all vehicles have or can obtain perfect information about the positions of all the other vehicles - if this is not the case, the approaches collapse. In a large scale dynamic network, this is not a realistic assumption since it would require a large scale communication intranet with frequent updates and interagent coordination and unnecessarily expose position information to those not part of the network.

Thus, if the early heuristic studies made us believe that a local approach was possible and the subsequent approaches actually proved convergence, the next step to take is to find a scheme that is actually consistent with large, fluid swarms and makes realistic assumptions about the information available to each vehicle about the system state at any time. Indeed, this is a difficult problem in which finding the appropriate control is quite a challenge; however, the reward of its solution will be a distributed approach that is directly applicable to the micro/nano-robotic hardware currently available.

Others have already attempted to find potential field schemes as an alternative to the perfect information approaches cited above, for problems ranging from more static single-vehicle settings[4] to the coordination of a network of vehicles.[12] In the former case, an electromagnet was used to guide an underwater vehicle to its static dock; in the latter case, inspired

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by insect navigation methods, artificial pheromones were introduced, the increasing or decaying quantities of which were stored in data bases and used for trajectory planning.

This paper investigates the feasibility of a total field sensing approach of magnetic nature as an alternative to approaches where each vehicle needs to know the positions of some or all of the other vehicles in order to plan a safe trajectory. In this setting, each vehicle generates a magnetic field around itself and is provided with a set of magnetic sensors that sense the total field generated by the whole network of vehicles. The vehicle's own field is static in its reference frame and known; however, the field contributions of the other vehicles are unknown and vary as the vehicles move. Since the magnetic flux density decays inversely proportional to the cube of the distance from the magnetic source, the contribution of vehicles farther away will be very small compared to that of vehicles situated closer to the sensing vehicle. The magnetic field generated by the sensing vehicle itself can nearly be cancelled out with sophisticated placement methods, leaving the the dynamic field generated by the other vehicles. In particular, by strategically placing one-axis magnetic sensors orthogonal to the vehicle's own field at different locations, the vehicle's own field will essentially be eliminated from the measurements. Furthermore, by choosing these locations in a symmetric way, one can also automatically cancel out the Earth's magnetic field component, thus making sure that the net difference only reflects the total field generated by the $N-1$ other vehicles. By combining the measurements of several on-board sensors placed at different locations, the vehicle can estimate the gradient of the total field generated by the other vehicles and move in the opposite direction to avoid collision. Thus, no vehicle has to know the position of any of the other vehicles in order to plan its trajectory.

Recent results in biology suggest that similar mechanisms may be found in natural systems such as flocks of birds, schools of fish and bee swarms.[3] Indeed, it has been verified that the navigation skills of such systems can be severely hampered by the application of disturbing magnetic fields. An interesting application where the total field sensing approach could be useful is robosoccer.[2]

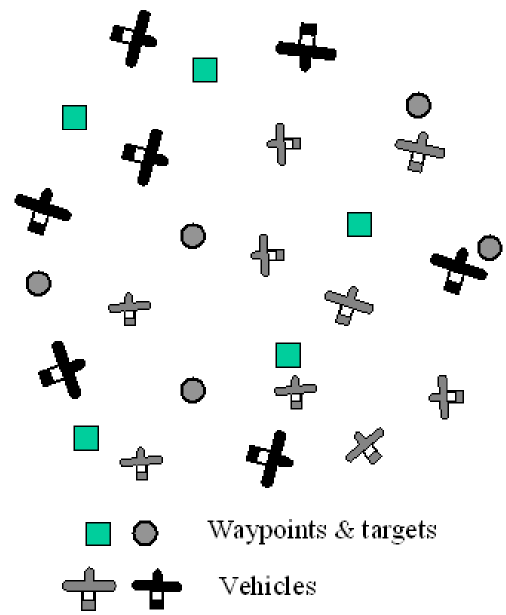

Fig. 1: Typical scenario for our problem, where a set of vehicles are matched to a set of waypoints to visit. How can one ensure the safe navigation of each vehicle to its waypoint(s) without collisions with any of the other vehicles?

Below, a formal problem statement is given first. The total field sensing approach is then presented and a brief review of magnetic field theory is given. After that, the implementation of our approach is described in detail and, finally, the theory is illustrated with simulation results.

PROBLEM STATEMENT

Assuming a system of N vehicles $k = 1, \ldots, N$, each associated at time $t = 0, 1, 2, \ldots$ with a position $\mathbf{r}^k(t)$, a start position $\mathbf{r}^k(0)$ and a goal position $\mathbf{r}^k_G$ in a common coordinate system R, the objective is for all vehicles to reach their respective goal positions within a common time limit $t \leq t_{\max}$ without colliding with any of the other vehicles. A typical scenario is shown in figure (1).

Why is this a hard problem? Firstly, it is a difficult problem from a control point of view since there are many degrees of freedom and the complexity grows exponentially with N. As noted above, this soon makes the problem untractable with centralized methods and a distributed control approach is necessary. This aspect of the problem is the one most often recognized as difficult in the previous literature - as noted above, the solutions proposed consisted in going from centralized control to local methods. These local methods were at first heuristic but have lately been complemented with analytical versions proven to converge.

Another difficult aspect of the problem has to do with information and has hitherto been given much less attention. While the distributed methods proposed in the last decade made the problem tractable, they kept the assumption of perfect information used in centralized control. However, such an assumption is unnatural in a distributed framework for several reasons. Firstly, such extreme redundancy of information goes against the distributed control paradigm itself - some information redundancy is indeed desired, but the duplication of all information is not. Secondly, perfect information in swarms is extremely difficult to implement with current sensing technology - estimating positions and velocities with machine vision and sonar is difficult even when N is small; as N grows, this becomes even harder since vehicles may block each other or distances to units farther away may be relatively large. Thirdly, an implementation would thus involve an extensive communication network which could be very costly and might expose information, thus constituting a security risk.

PROPOSED APPROACH

This section briefly introduces the proposed algorithm along with a short review of magnetic field theory. This is followed with a more detailed description and motivation of the approach, and comments on a number of hardware issues.

As noted above, centralized approaches to the problem rapidly become intractable as the number of vehicles N grows and it is therefore necessary to opt for a decentralized approach, making each vehicle responsible for its own trajectory planning. Ideally, each vehicle would be able to navigate along a straight line from its start position to its goal without colliding with other vehicles - our approach consists in superposing on such a naive scheme a collision avoidance algorithm based on total field sensing which makes information about the particular positions and velocities of the other vehicles unnecessary. Since this is a highly dynamic form of potential field, we first briefly review the theory of artificial potential fields in trajectory planning.

Background: Potential Fields – The potential field approach for obstacle avoidance, proposed by Khatib in 1986,[6] models goals and obstacles as attractors and repellers of an artificial potential field in which each vehicle moves. The great advantage of this approach was its tractability, which allowed highly adaptive on-line trajectory planning instead of the rigid, time-consuming but exact off-line approaches in use at the time. The most serious shortcoming of the approach was the risk of local minima and of oscillations near obstacles and in narrow passages.[7]

The theoretical simplicity of the potential field approach comes at the price of assuming perfect information or perfect extraction of information from the sensors. In the former case, the navigating vehicle has a map, knows the positions of all the obstacles relative to itself and can thus inversely deduce the forces that it would sense. In the latter case, the vehicle has sensors good enough to perfectly estimate the positions and velocities of the obstacles - the sensing modes traditionally suggested are machine vision or sonar - and then again deduce the forces. The first case implies a static situation with no real need for on-line path planning whereas in the second case, unrealistic assumptions are made on the sensors. In both cases, the unnatural detour needed to actually obtain the fictitious forces goes against the simplicity of the theoretical framework.

Is it possible to try other sensing modes to actually sense real forces, thus giving the potential field approach a simplicity in implementation to match that of the theoretical framework? Could the sensing of such forces be made more accurate than the estimation of distances and velocities based on visual or sonar input?

Superposing Total Field Collision Avoidance – When $N = 1$, there is no uncertainty in the system and the single vehicle can navigate along the straight line from $\mathbf{r}^1(0)$ to $\mathbf{r}^1_G$. However, as soon as $N > 1$, such simple paths may no longer be safe and detours will have to be planned around detected vehicles. In addition, as the number of vehicles N increases, the uncertainty grows exponentially and the task of keeping track of the exact positions of all or even a fraction of the other vehicles becomes overwhelming - rather, one will have to focus on whether any of the other vehicles is too close.

Magnetic Force – We thus want to find a common signal, preferably in the form of a real force, that can be readily measured and informs each vehicle if any of the other vehicles is close and in which direction it is approaching. Gravity and electromagnetic forces all decay with the distance between the source and the sensor, which is desirable in our case. While gravity and electrostatic forces seem inappropriate due to weak signals and lack of suitable sensors, respectively, magnetic forces can conveniently be measured with a range of commercial sensors. Recent results in biology suggest that magnetic senses are important navigation tools in a wide range of animals moving collectively in swarms, herds or schools.[3]

With this in mind, we propose a **total field** collision avoidance scheme of magnetic nature which requires each vehicle to generate a local magnetic field with an on-board magnet and sense the total surrounding field with a set of magnetic sensors.

Background: Magnetic Field Theory

Charged particles in motion generate magnetic fields - here, we review the central notions in magnetic field theory and examine the flux density around current-carrying conductors in general and the magnetic dipole in particular.

Definitions – The two central notions in magnetic field theory are the magnetic flux density B and the magnetic field intensity H, both vector quantities. We will deal with the magnetic flux density B, given in Teslas, T, or in Gauss, 1 Gauss=10^{-4} T - the magnetic field of Earth corresponds at its surface to a magnetic flux density component of about 0.5 Gauss.

Current-Carrying Conductor – The magnetic flux density around an infinitely long circular current-carrying conductor forms concentric circles in any plane perpendicular to the direction of the current. When the conductor has finite length and forms a closed loop, we obtain what is referred to as a magnetic dipole.

Magnetic Dipole – The magnetic flux density **B** around a magnetic dipole - a small current-carrying loop -

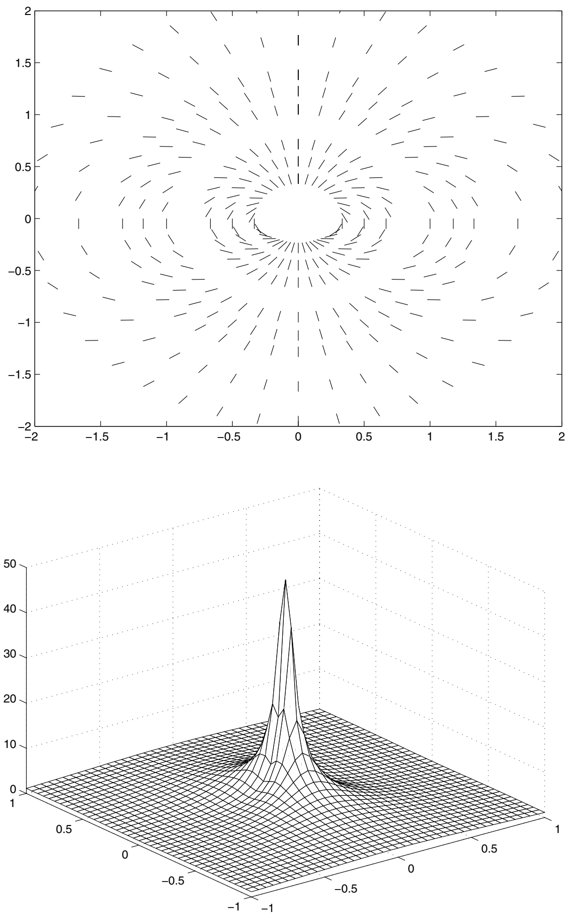

Fig. 2: Magnetic flux density $\frac{B}{|B|}$ in the xy-plane around a magnetic dipole, above, and $|B|$, below.

can be described as

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3}(\mathbf{e}_r 2\cos\theta + \mathbf{e}_\theta \sin\theta), \qquad (1)$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the permeability of free space, $m\mathbf{e}_y$ is the magnetic dipole moment and $(r,\, \theta)$ are the spherical coordinates in a coordinate system - the dipole is situated at its origin.[1,9] The vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the orthogonal unit vectors in the spherical coordinate system. Importantly, the magnetic flux lines around a bar magnet can be very well approximated by those generated by a magnetic dipole.

Vehicles as Magnetic Dipoles

By providing each vehicle with a magnet, we transform our system of navigating vehicles into a system of mobile magnetic dipoles that can be sensed by each vehicle.

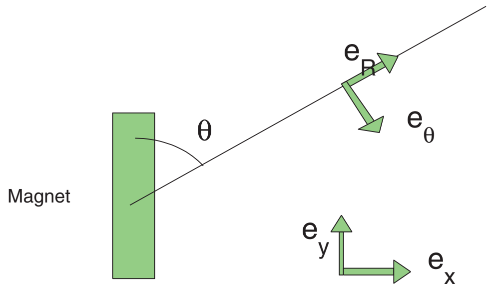

Fig. 3: Spherical coordinate axes and Cartesian coordinate axes.

Generating the Local Field

Each vehicle k carries a magnet of moment m on board that generates a magnetic field $\mathbf{B}^k(\cdot, t)$. Although constant in the local reference frame R^k of vehicle k, the magnetic field $\mathbf{B}^k(\cdot, t)$ will vary in the common reference frame R as vehicle k moves. By the analogy with a magnetic dipole described above, each magnet naturally defines a local spherical coordinate system with the magnet at the origin. At any point j described in R^k by the coordinates (r_k^j, θ_k^j), the field $\mathbf{B}^k(j,\cdot)$ generated by vehicle k will be

$$\mathbf{B}^k(j, \cdot) = \frac{\mu_0 m}{4\pi (r_k^j)^3} (\mathbf{e}_{r_k^j} 2\cos\theta_k^j + \mathbf{e}_{\theta_k^j} \sin\theta_k^j). \quad (2)$$

Sensing the Total Field

In addition, each vehicle has a set of magnetic field sensors on board with which it can sense the total magnetic field at the corresponding set of points and draw conclusions about the gradient of the total magnetic field. From above, we have that the total field sensed at a point j by any vehicle will be

$$\mathbf{B}(j, \cdot) = \sum_{k=1}^{N} \frac{\mu_0 m}{4\pi (r_k^j)^3} (\mathbf{e}_{r_k^j} 2\cos\theta_k^j + \mathbf{e}_{\theta_k^j} \sin\theta_k^j). \quad (3)$$

As we can see, the cubic factor in the denominators above will make the vehicle's own magnetic field the dominating term in the sum above when j is the position of an on-board sensor.

Subtracting Own Field

Since the vehicle's own magnetic field is constant and known, ideal sensors would make it possible to subtract this field and obtain the net total field generated by the $N-1$ other vehicles. Each vehicle k would then sense the other vehicles at point j as $\mathbf{B}_S^k(j)$, where

$$\mathbf{B}_S^k(j, \cdot) = \sum_{l \neq k} \frac{\mu_0 m}{4\pi (r_l^j)^3} (\mathbf{e}_{r_l^j} 2\cos\theta_l^j + \mathbf{e}_{\theta_l^j} \sin\theta_l^j). \quad (4)$$

Certain commercial sensors have patented offset straps which make it possible to apply a magnetic field in the direction opposite to the external magnetic field, thus cancelling out the external field and making sure that the net field stays in the range within which the sensing is linear. This thus permits a more sophisticated form of subtraction of the own field.

Strategic Sensor Positions

Due to hardware limitations and the difference in magnitude between the vehicle's own field and that generated by the other vehicles, a straightforward subtraction as described above may not always be possible. However, using the fact that the vehicle's own field is constant and known both in magnitude and direction in the vehicle's own reference frame, one can make the described subtraction unnecessary by strategically positioning the sensors to measure the total field orthogonal to the vehicle's own field at different points.

As noted above, at a given on-board point j the sensor senses the vehicle's own field as $\mathbf{B}^k(j,\cdot)$, where r_j^k is the distance from the magnet to the sensor and θ_j^k the angle between the axis of the magnet and the vector from the magnet to the sensor:

$$\mathbf{B}^k(j, \cdot) = \frac{\mu_0 m}{4\pi (r_k^j)^3} (\mathbf{e}_{r_k^j} 2\cos\theta_k^j + \mathbf{e}_{\theta_k^j} \sin\theta_k^j). \quad (5)$$

Since all magnetic field lines are closed and continuous, it is possible to find exact angles θ_j^k where the vehicle's own field is respectively parallel and orthogonal to the axis of the vehicle's magnet. The former case is the easier one: when θ_j^k is respectively 0, $\frac{\pi}{2}$ or π, the vehicle's own field is parallel to the magnet axis. The latter case requires some calculations and leads to the values for θ_j^k of $\arccos\frac{1}{\sqrt{3}}$ or $(\pi - \arccos\frac{1}{\sqrt{3}})$, that is, 54.7^o or 125.3^o.

To prove this, we first express the spherical unit vectors $\mathbf{e}_R$ and $\mathbf{e}_\theta$ in the Cartesian unit vectors $\mathbf{e}_x$ and $\mathbf{e}_y$, where $\mathbf{e}_y$ is parallel to the magnet axis, as shown in figure (3). Note that these expressions are different from those used to express the polar unit vectors in the Cartesian coordinates. We thus obtain, with the sign depending on the quadrant:

$$\mathbf{e}_R = \pm \sin\theta \mathbf{e}_x + \cos\theta \mathbf{e}_y \tag{6}$$

$$\mathbf{e}_\theta = \pm \cos\theta \mathbf{e}_x - \sin\theta \mathbf{e}_y \tag{7}$$

Using these relations gives

$$\begin{aligned} 2\cos\theta \mathbf{e}_R \; &+ \; \sin\theta \mathbf{e}_\theta \\ &= \; 2\cos\theta(\pm\sin\theta\mathbf{e}_x + \cos\theta\mathbf{e}_y) \\ &\quad + \sin\theta(\pm\cos\theta\mathbf{e}_x - \sin\theta\mathbf{e}_y) \qquad (8) \\ &= \; \pm 3\cos\theta\sin\theta\mathbf{e}_x + (2\cos^2\theta - \sin^2\theta)\mathbf{e}_y \end{aligned}$$

For the first case, we want the $\mathbf{e}_x$-component of the field, $\pm 3\cos\theta\sin\theta$, to be zero. We have:

$$\begin{aligned} \pm 3\cos\theta\sin\theta = 0 \;&\Rightarrow\; \cos\theta = 0 \text{ or } \sin\theta = 0 \\ &\Rightarrow\; \theta = 0, \frac{\pi}{2}, \pi. \end{aligned} \tag{9}$$

In the second case, we want the $\mathbf{e}_y$-component,

$$2\cos^2\theta - \sin^2\theta$$

to vanish. Using basic trigonometric relations gives

$$\begin{aligned} 2\cos^2\theta \;&-\; \sin^2\theta \\ &= \; 2\cos^2\theta - 1 + \cos^2\theta = 3\cos^2\theta - 1 = 0 \\ &\Rightarrow\; \cos\theta = \frac{\pm 1}{\sqrt{3}} \Rightarrow \theta = \arccos\frac{\pm 1}{\sqrt{3}}. \qquad (10) \end{aligned}$$

We have now found a set $\Theta_x = \{0, \frac{\pi}{2}, \pi\}$ of angles where the vehicle's own field is orthogonal to the x-axis in the vehicle's reference frame and a set of angles $\Theta_y = \{\arccos\frac{1}{\sqrt{3}}, (\pi - \arccos\frac{1}{\sqrt{3}})\}$ where, likewise, the vehicle's own field is orthogonal to the y-axis in the vehicle's reference frame; these angles are shown in figure (4). Thus, if a field component along the x-axis is detected at an angle $\theta_x \in \Theta_x$ or a field component along the y-axis is measured at an angle $\theta_y \in \Theta_y$, we know that they were not generated by the vehicle itself.

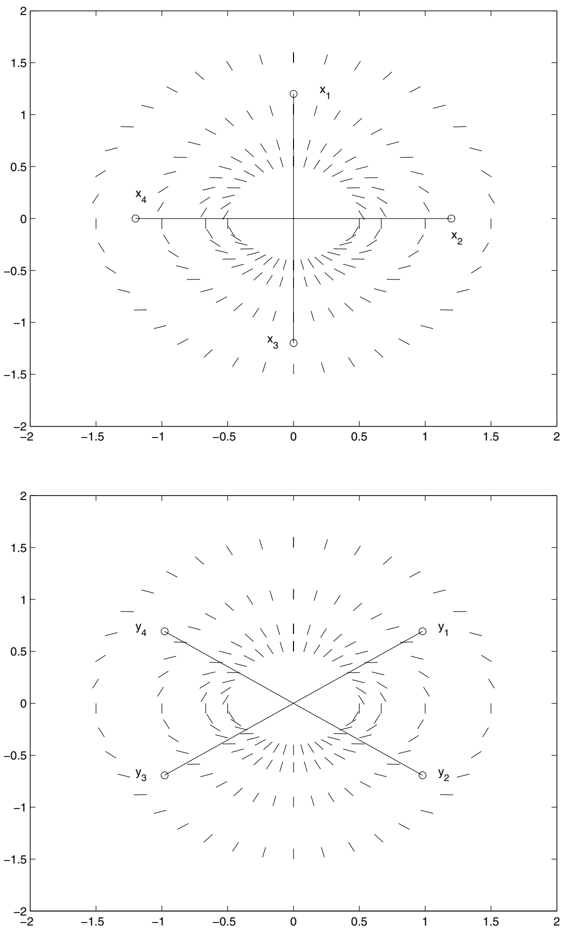

Fig. 4: Sensor input positions for measuring the surrounding field along the x-axis, above, and along the y-axis, below.

Earth's Magnetic Field

The magnetic field of Earth is generated by magnetic material in the deep interior of the planet. At any point on the Earth's surface, the field has a component in the tangent plane directed towards the magnetic South pole, close to the geographical North pole, and an orthogonal component which we disregard in a two-dimensional setting.

While the strategic sensor positions described above ensure that the vehicle's sensors will not sense its own field, the total field sensed will in general include a component of the magnetic field of the Earth - as the vehicles rotate, this component will change. However, the symmetric choice of the strategic sensor positions as illustrated above will make sure that the component $\mathbf{B}_x$ of the Earth's magnetic field sensed by any of the sensors positioned to measure the x-component will be identical, and the same is true for the component $\mathbf{B}_y$ sensed by the sensors positioned to measure the y-component. Thus, the pairwise subtractions of the measurements of x-sensing sensors or of y-sensing sensors will always cancel out the Earth magnetic field component.

Why will the magnetic field component of Earth measured by the sensors be the same while that is not true for the measurements of the total field generated by the $N-1$ other vehicles? That is because the distance to the magnetic field source of Earth is infinitely much larger than the distances to the other vehicles, so that the on-board sensors of a vehicle are all at essentially the same distance from the interior of Earth whereas their respective distances to the vehicles vary and make a difference.

Vehicle Navigation

In order to navigate safely, each vehicle needs to use the sensor measurements to estimate the gradient of the total field and use this for collision avoidance. The collision avoidance and the navigation towards the goal then need to be weighted to obtain the desired net result of safe, yet aggressive navigation towards the goal. The navigation of each vehicle k is expressed by the following equation, where the difference $\Delta\mathbf{r}^k(t)$ is the weighted sum of a collision avoidance component $\Delta\mathbf{r}_{coll}$ and a component of naive navigation straight towards the goal $\Delta\mathbf{r}_{goal}$:

$$\mathbf{r}^k(t+1) = \mathbf{r}^k(t) + \Delta\mathbf{r}^k(t). \tag{11}$$

Navigating towards the Goal

Safe navigation towards the goal is achieved as the weighted sum of collision avoidance and moving along the straight line towards the goal. We have for some weight γ, $0 < \gamma < 1$:

$$\Delta\mathbf{r}_{goal} = \gamma\frac{\mathbf{r_G} - \mathbf{r}}{|\mathbf{r_G} - \mathbf{r}|} \tag{12}$$

Collision Avoidance

The collision avoidance component $\Delta\mathbf{r}_{coll}$ is parallel to the estimated gradient of the total field ∇B_{est} but of opposite direction. We thus have:

$$\Delta\mathbf{r}_{coll} = -(1-\gamma)\frac{\nabla B_{est}}{|\nabla B_{est}|}. \tag{13}$$

Estimating the Gradient

Each vehicle has eight single-axis sensors positioned at the eight strategic positions chosen above and shown in figure (4). The four sensors positioned to sense the x-component of the surrounding magnetic field are numbered clockwise and give the inputs x_i, $i = 1,2,3,4$ corresponding to the spherical angles 0^o, 90^o and 180^o. The same is true for the four sensors measuring the y-component as inputs y_i, $i = 1,2,3,4$; they correspond to the spherical angles 54.7^o and 125.3^o. We wish to estimate the gradient

$$\nabla B = \frac{\partial}{\partial x}B\mathbf{e_x} + \frac{\partial}{\partial y}B\mathbf{e_y}$$

at the origin of the vehicle's own reference frame. We have

$$\begin{aligned}\frac{\partial}{\partial x}B &= \frac{\partial}{\partial x}\sqrt{B_x^2 + B_y^2} \\ &= \frac{1}{\sqrt{B_x^2 + B_y^2}}(B_x\frac{\partial B_x}{\partial x} + B_y\frac{\partial B_y}{\partial x})\end{aligned} \tag{14}$$

and

$$\begin{aligned}\frac{\partial}{\partial y}B &= \frac{\partial}{\partial y}\sqrt{B_x^2 + B_y^2} \\ &= \frac{1}{\sqrt{B_x^2 + B_y^2}}(B_x\frac{\partial B_x}{\partial y} + B_y\frac{\partial B_y}{\partial y}).\end{aligned} \tag{15}$$

We approximate

$$\frac{\partial B_x}{\partial x} \approx \frac{x_2 - x_4}{l_{x_{24}}} \tag{16}$$

and

$$\frac{\partial B_x}{\partial y} \approx \frac{x_1 - x_3}{l_{x_{13}}}, \tag{17}$$

where $l_{x_{ij}}$ is the distance between the sensors x_i and x_j. We average to approximate

$$\frac{\partial B_y}{\partial x} \approx \frac{y_1 - y_4}{2l_{y_{14}}} + \frac{y_2 - y_3}{2l_{y_{23}}} \quad \text{and} \tag{18}$$

$$\frac{\partial B_y}{\partial y} \approx \frac{y_4 - y_3}{2l_{y_{43}}} + \frac{y_1 - y_2}{2l_{y_{12}}}, \tag{19}$$

where $l_{y_{ij}}$ is the distance between the sensors y_i and y_j. The values B_x and B_y are estimated from the sensor measurements as

$$B_x \approx \frac{x_1 + x_2 + x_3 + x_4}{4} \tag{20}$$

and

$$B_y \approx \frac{y_1 + y_2 + y_3 + y_4}{4}. \tag{21}$$

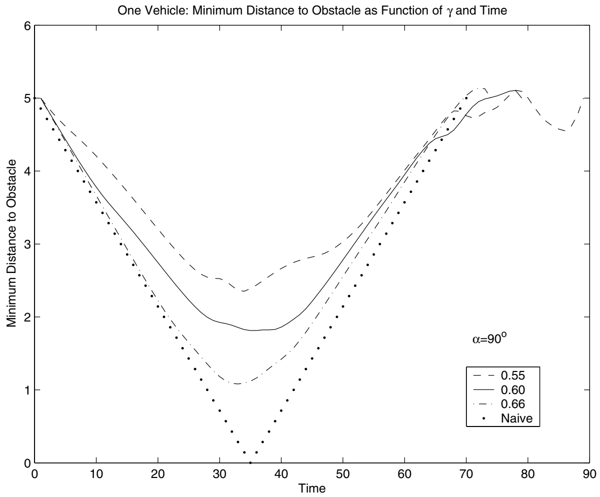

Fig. 5: Variation with γ and of minimum distance between navigating vehicle and static vehicle.

The estimate ∇B_{est} of the gradient thus obtained is used to generate a collision avoidance component in the motion of the vehicle. Using the approximations for B_x and B_y in (20) and (21), gives

$$\begin{aligned} \nabla B_{est} &= \frac{1}{\sqrt{B_x^2 + B_y^2}} \left[B_x \frac{x_2 - x_4}{l_{24}} \right. \\ &\left. + B_y \left(\frac{y_1 - y_4}{2l_{y_{14}}} + \frac{y_2 - y_3}{2l_{y_{23}}} \right) \right] \mathbf{e_x} \\ &+ \frac{1}{\sqrt{B_x^2 + B_y^2}} \left(B_x \frac{x_1 - x_3}{l_{13}} \right. \\ &\left. + B_y \left(\frac{y_4 - y_3}{2l_{y_{43}}} + \frac{y_1 - y_2}{2l_{y_{12}}} \right) \right] \mathbf{e_y} \quad (22) \end{aligned}$$

Net Motion

A third important factor was the weighting between the two possibly conflicting objectives of orientation towards a goal and collision avoidance, respectively. We wanted to achieve aggressive path planning and got the best results with a weight of $\gamma = 0.66$ on the orientation towards the goal and $(1 - \gamma) = 0.34$ on collision avoidance. The net motion is given by $\Delta\mathbf{r}$

$$\begin{aligned} \Delta\mathbf{r} &= \Delta\mathbf{r}_{goal} + \Delta\mathbf{r}_{coll} \\ &= \gamma \frac{\mathbf{r_G} - \mathbf{r}}{|\mathbf{r_G} - \mathbf{r}|} - (1 - \gamma) \frac{\nabla B_{est}}{|\nabla B_{est}|} \quad (23) \end{aligned}$$

Figure (5) shows the variation of the minimum distance between a navigating vehicle and a second static vehicle as a function of γ.

IMPLEMENTATION

Our algorithm was tested in a set of simulations which proved the algorithm to give safe, yet aggressive navigation. We also performed a series of measurements with actual magnetic sensors and magnets to make sure that the theoretical assumptions forming the basis for our algorithm were reasonable and accurate. Below, we will first present our simulations and then describe our hardware experiments.

Simulation Results

This section presents the simulation scenario used to test the algorithm in a highly cluttered and constrained space. In this scenario, ten vehicles were assigned start and goal positions from the sets S and G of start and goal positions, respectively, where $S = \{(i, 0) \mid i = 0, 1, ..., 9\}$ and $G = \{(i, 10) \mid i = 0, 1, ..., 9\}$. All vehicles started at different points $s \in S$ and had different goal points $g \in G$. A naive scheme of heading straight towards the goals without collision avoidance would result in numerous collisions; however, with our approach there were no collisions, although the resulting trajectories are quite aggressive. With different magnet orientations, it was possible to obtain more defensive trajectories.

The minimum distance between any pair of vehicles was plotted as a function of time for the total field approach with $\gamma = 0.66$ - the result is shown in figure (6). While the approach with no collision avoidance would have led to several collisions, the total field approach ensured that each vehicle kept a minimum safety distance to all other vehicles. The approach was also compared to a standard potential field algorithm where the repulsive potential of the other vehicles was modelled as being inversely proportional to the square of the distance to the sensing vehicle (rather than the cube in the magnetic case). Our experiments for these simulations indicated that the potential field method was very sensitive to local minima.

Figures (7), (8) and (9) compare the various trajectories generated by the total field approach and with the naive scheme without collision avoidance, respectively. The positions of all ten vehicles are marked with the symbols $\circ$, $\star$ or Δ at several interesting points along the trajectories; at these points in time, collisions would have occurred with the naive scheme.

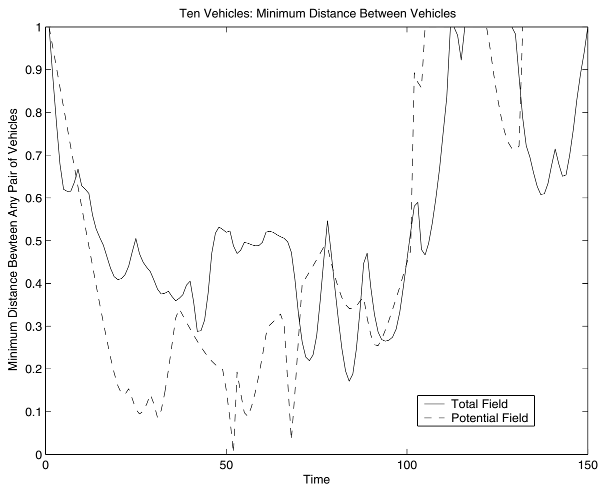

Fig. 6: Minimum distance between any vehicle pair, with the total field approach ($\gamma = 0.66$) and the potential field method.

Hardware Implementation

We also performed experiments with hardware using one sensor and two identical light-weight magnets to simulate the scenario of two vehicles moving relative to each other. Furthermore, a series of measurements was carried out to confirm the theoretical deductions such as the strategic sensor positions and detection ranges and to test the function of the patented offset straps of the sensor - for the latter measurement series, we used two sensors and a set of magnets of varying sizes.

The sensors used were Honeywell's HMC2003 and HMR2300, which are three-axis magnetic sensors targeted at fields up to 2 Gauss and have a resolution of less than 70 μGauss. The sensors are equipped with patented offset straps that allow the application of an electrically generated magnetic field in a direction opposite to the exterior magnetic field, thus making it possible to locally cancel out any component of the exterior field and bring the net field back to the linear range. The magnets used to simulate two navigating vehicles were two identical, cylindrical magnets of length 18 mm, diameter 5 mm and weight 6 grams - their magnetic moments were aligned with the cylinder axes.

In the measurements to confirm the theoretical deductions, we verified the ratio between the detection range and the on-board distance between the vehicle's own sensors and magnet to be around thirty. This was also obtained theoretically by defining the on-board distance between the vehicle's own sensors and magnet as the distance where the sensors would measure 2 Gauss and the detection range as the distance where the sensing vehicle would measure less than 70μGauss. We also verified the strategic sensor positions to correspond to those calculated above. The detection ranges for the different magnets tested were found to vary from 1m to 7m.

In our simulation of a two-vehicle scenario, we positioned the sensor at a strategic position in Θ_x or Θ_y close to one of the magnets, thus simulating a vehicle with an on-board magnet and one sensor. The sensor and the first magnet were kept static throughout the test. The other magnet was mobile and used to simulate a second vehicle moving relative to the first one. The same sets of tests were performed once for Θ_x and once for Θ_y. We let the mobile vehicle travel along the x-axis and y-axis, respectively, first approaching and then passing the static vehicle. In this scenario, we got interference from the on-board magnet, situated as close as 1-2 cm to the sensor - this reduced the detection range ratio from thirty to fifteen, likely because the sensors used - HMC1002 as part of HMR2300 - do not compensate for cross-axis effects, as do more advanced sensors.

CONCLUSIONS

This paper investigated the feasibility of a novel approach to collision avoidance in the collective navigation of many vehicles in constrained spaces. In this total field sensing approach, each vehicle is modelled as a magnetic dipole and provided with sensors to sense the total field. By placing single-axis magnetic sensors orthogonal to the field generated by the vehicle itself, it was possible to measure the field generated by the other vehicles and estimate its gradient. The estimate could then be used in order to move in the direction opposite to the gradient, thus avoiding collision with other vehicles. This allowed safe, yet aggressive navigation without requiring each vehicle to know the positions of any of the other vehicles. This collision avoidance module was then combined with a naive motion along a straight line towards the goal to obtain safe decentralized navigation of a set of vehicles towards their respective goals with realis-

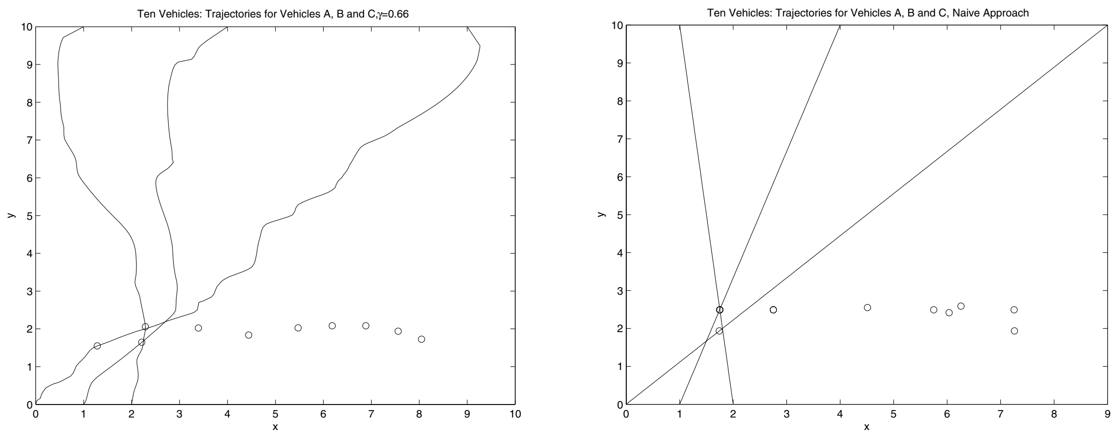

Fig. 7: Safe aggressive trajectories for vehicles A, B and C, to the left; trajectories without collision avoidance to the right. The positions of all vehicles at a particular time are indicated by o.

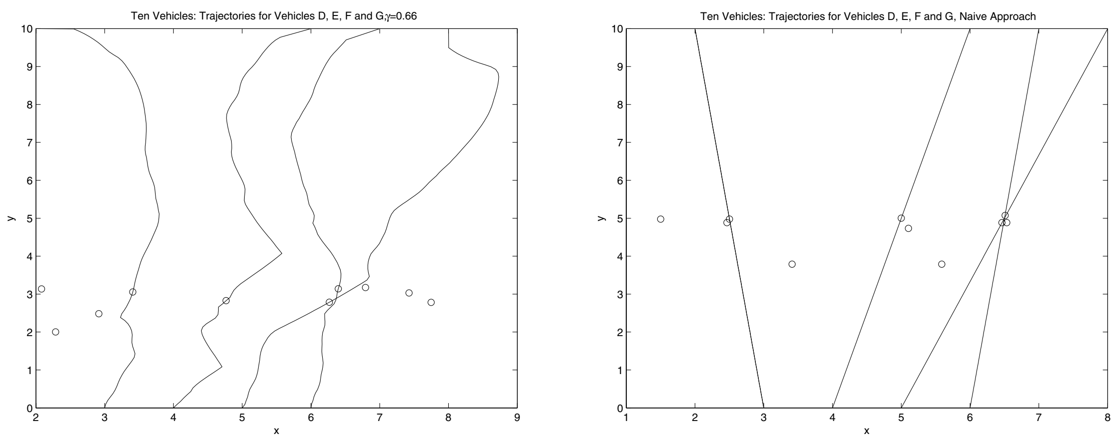

Fig. 8: Safe aggressive trajectories for vehicles D, E, F and G, to the left; trajectories without collision avoidance to the right. The positions of all vehicles at a particular time are indicated by o.

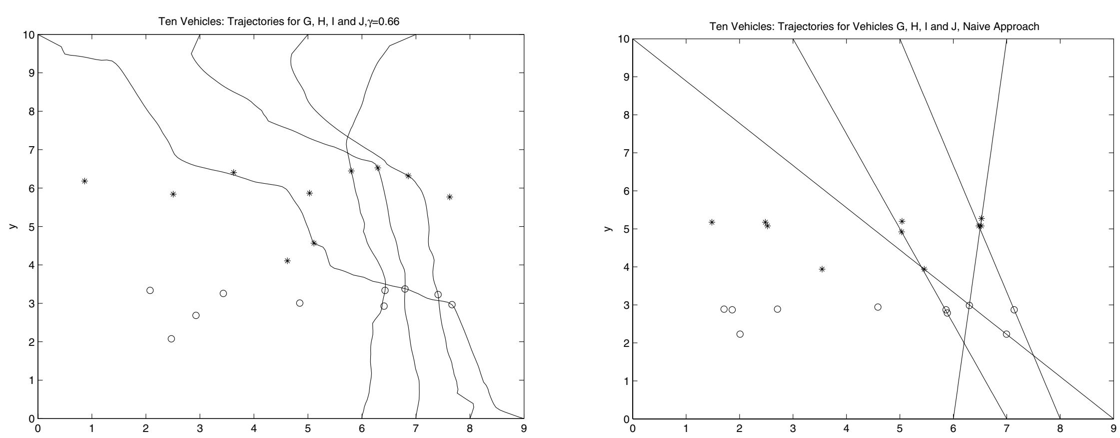

Fig. 9: Safe aggressive trajectories for vehicles G, H, I and J, to the left; trajectories without collision avoidance to the right. The positions of all vehicles at two particular times are indicated by o and *, respectively.

tic assumptions about the information each vehicle has about the other vehicles. While further work would be required to apply this technique to a real system, it appears to be a promising alternative to the current approaches which assume that perfect information is shared between all vehicles.

ACKNOWLEDGMENTS

Research funded in part under AFOSR grant F49620-01-1-0453.

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