Cooperative Path Planning for Multiple UAVs in Dynamic and Uncertain Environments [1]

John S. Bellingham, Michael Tillerson, Mehdi Alighanbari[2], and Jonathan P. How[3]
MIT Department of Aeronautics and Astronautics

Abstract

This paper addresses the problem of cooperative path planning for a fleet of UAVs. The paths are optimized to account for uncertainty/adversaries in the environment by modeling the probability of UAV loss. The approach extends prior work by coupling the failure probabilities for each UAV to the selected missions for all other UAVs. In order to maximize the expected mission score, this stochastic formulation designs coordination plans that optimally exploit the coupling effects of cooperation between UAVs to improve survival probabilities. This allocation is shown to recover real-world air operations planning strategies, and to provide significant improvements over approaches that do not correctly account for UAV attrition. The algorithm is implemented in an approximate decomposition approach that uses straight-line paths to estimate the time-of-flight and risk for each mission. The task allocation for the UAVs is then posed as a mixed-integer linear program (MILP) that can be solved using CPLEX.

1 Introduction

The capabilities and roles of UAVs are evolving and require new concepts for their control[1, 2]. For example, today's UAVs typically require several operators, but future UAVs will be designed to make tactical decisions autonomously and will be integrated into teams that cooperate to achieve high-level goals, thereby allowing one operator to control a fleet of UAVs. New methods in planning and execution are required to coordinate the operation of these fleets.

Real-world air operations planners employ cooperation between aircraft in order to manage the risk of attrition. Missions are scheduled so that one group of aircraft opens a corridor through anti-aircraft defenses before a follow-on group attacks higher value targets, preserving their survival. When each UAV can perform multiple functions (*e.g.,* both destroy anti-aircraft defenses and attack high value targets) it is very challenging to plan missions to exploit the integrated capabilities of the team. Note that cooperation is not just desirable; it is crucial for designing successful missions in heavily defended environments. A successful method of performing the allocation cannot simply assume the mission will always be executed as designed, given an adversary in the environment who is actively attempting to cause failure. Simulations are presented to show that ignoring the probability of UAV loss results in mission plans that are quite likely to fail. Furthermore, techniques that model this probability [8, 9], but ignore its coupling to each UAV's mission can result in very poor performance of the fleet.

Clearly, a UAV mission planning formulation must recognize the importance of managing UAV attribution, and have the capability to use the same strategies as real-world air operations planners. The new formulation in this paper approaches this by capturing not only the value of the waypoints that each UAV visits and of returning the UAV safely to its base, but also by capturing the probability of these events. In order to maximize mission score as an expectation, this stochastic formulation designs coordination plans that optimally exploit the coupling effects of cooperation between UAVs to improve survival probabilities. This allocation is shown to recover planning strategies for air operations and to provide significant improvements over prior approaches [8, 9]. The paper briefly presents the decomposition method for solving the UAV coordination and control problem. It is then shown how to extend that formulation to capture the stochastic effects of the environment. Three solution methods are discussed and then compared on a simulation example.

The optimal fleet coordination problem includes team composition and goal assignment, resource allocation, and trajectory optimization. These are complicated optimization problems for scenarios with many UAVs, obstacles, and targets. Furthermore, these problems are strongly coupled, and optimal coordination plans cannot be achieved if this coupling is ignored [4, 5]. Figure 1 shows an approximate method for solving the UAV coordination and control problems, which offers much faster solution times, but could yield sub-optimal results [4]. The cost function used is the overall mission completion time plus a small weighting on the individual UAV finishing times. The costs are estimated based on the finishing times found using straight-line path approximations. Note that significant pruning of

[1] Funded by MICA DARPA contract N6601-01-C-8075.
[2] Research Assistant, **mehdi_a@mit.edu**
[3] Associate Professor, **jhow@mit.edu**

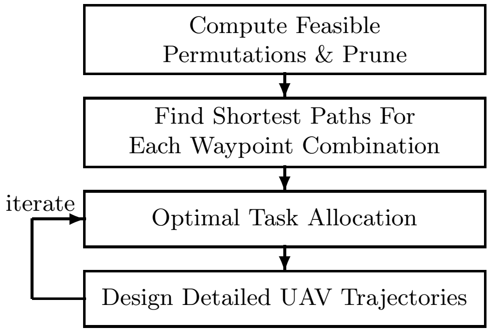

Fig. 1: Steps in decomposition algorithm [4].

the possible mission scenarios can be performed at several stages of the algorithm to reduce the size of the task optimization problem.

With these approximate finishing times available, the task assignment problem can be performed to minimize the approximate cost, which is posed as a MILP optimization and solved using CPLEX. The objective is to assign a permutation to each UAV that is combined into the mission plan such that the cost of the mission is minimized and the waypoints visited (of N_W) meet the constraints. Defining $\bar{t} = \max_{p \in \mathcal{V}} t_p$, the problem is given by

$$\min \; J_1 \;=\; \bar{t} + \frac{\alpha}{N_V} \sum_{j \in \mathcal{M}} C_j x_j \qquad (1)$$

$$\text{s.t. } \forall i \in \mathcal{W} : \sum_{j \in \mathcal{M}} V_{ij} x_j \;=\; 1, \quad \forall p \in \mathcal{V} : \sum_{j \in \mathcal{M}_p} x_j = 1$$

where $\mathcal{V} = \{1, \ldots, N_V\}$ and N_V is the number of UAVs, $\mathcal{M} = \{1, \ldots, N_M\}$ is the set of all permutations and $\mathcal{M}_p \subseteq \mathcal{M}$ are the permutations that involve UAV p, $\mathcal{W} = \{1, \ldots, N_W\}$, $V_{\text{ij}} = 1$ if waypoint i is visited on the j^{th} permutation and 0 otherwise, and C_j is the cost (time) associated with the j^{th} permutation. The binary decision variable $x_j = 1$ if permutation j is selected, and 0 otherwise. The first constraint enforces that waypoint i is visited once. The second constraint prevents more than one permutation being assigned to each UAV. Note that relative timing constraints can also be included to enforce time intervals between the various events/visits [4]. The final step in the algorithm uses receding horizon [7] or fixed-assignment [5, 6] MILP methods to plan detailed trajectory commands for each UAV while accounting for the dynamics and inter-vehicle collision avoidance.

2 Planning and Re-planning Algorithms

The task allocation problem in the previous section is used to assign a sub-team of UAVs to visit a set of waypoints based on the information (UAV states, waypoint locations, and obstacles) known at the beginning of the mission. However, throughout the execution of the mission the environment and fleet can (and most likely will) change. As a result, the optimal allocation of the tasks amongst the UAVs in the fleet could be dramatically altered. Note that if the problem size is sufficiently small, it would be possible to perform a complete re-calculation of the task allocation problem using a new set of costs based on the updated environment. However, for larger problems it might be necessary to re-solve smaller parts of the allocation problem. Two smaller problems are presented in this section. One is a *local repair* where only one UAV assignment is altered. Another is a *sub-team allocation problem* where only those "directly influenced" by the change in environment are re-assigned.

2.1 Addition of Waypoint

As the mission is executed, it is possible that further reconnaissance will identify a new waypoint. In the *local repair* method the cost of adding the new waypoint to each UAV is determined using the UAV's current state, the remaining assigned waypoints from the original problem, and the new waypoint. The assignment of this waypoint that results in the smallest increase in the cost function is then chosen. The local repair can be solved very quickly, but it is a sub-optimal solution because it does not allow the UAVs to trade previously assigned waypoints.

A *sub-team* problem can be formulated which only considers those UAVs capable of visiting the new waypoint. These UAVs, their previously assigned waypoints, and the new waypoint are then considered as a smaller task allocation problem. This allows any waypoints within this group to be traded amongst the UAVs and avoids many of the limitations of the local repair method, but it is still sub-optimal. The optimal solution can be obtained by solving full re-allocation problem, but this optimization takes longer to compute.

2.2 Loss of UAV

Another possible change is a loss of a UAV in a fleet. In this case the waypoints assigned to that UAV must be re-assigned across the fleet. The reallocation can be formed through a local repair, sub-team re-allocation, or full re-allocation, as described above.

2.3 Addition/Removal of Obstacle

The addition or removal of an obstacle is considered by estimating the new cost for each UAV given their current assigned waypoints with (and without) the obstacle in question. If the UAV's cost estimate changes, then that UAV is considered to be influenced by the obstacle. The UAVs and their previously assigned waypoints are grouped into a new allocation problem and the re-assignment is performed for this subset of the fleet. If the UAV is influenced by the obstacle, the *local repair* method does not change its assignment of waypoints, but redesigns the detailed trajectory to account for the change. The *sub-team* problem considers

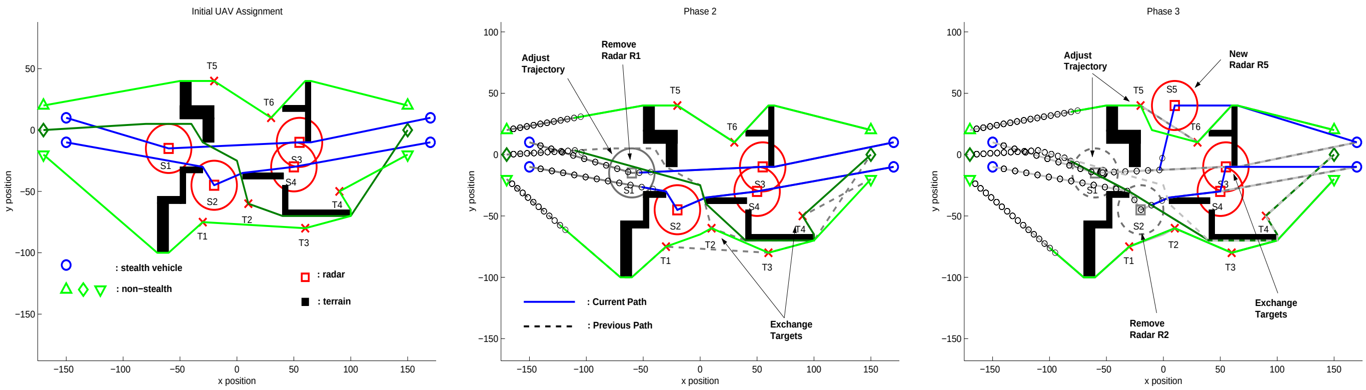

Fig. 2: Dynamic Environment Simulation. The stealth UAVs, ○ are capable of flying through and removing radar sites noted with □ 's. The other UAVs are restricted from flying through radar zones and must remove targets marked with ×'s.

all UAVs that are influenced by the obstacle.

2.4 Dynamic Environment Simulation

Figure 2 shows three snapshots of the simulation that applies the approximate allocation method presented in [4] to a dynamic environment. The changes in the environment during the simulation occur with no prior knowledge and the fleet is re-assigned through the sub-team re-allocation method.

The simulation includes two types of UAVs and two types of obstacles. The solid black areas are no fly zones that no UAV can pass through, such as mountains or buildings. The second obstacle type, marked with □, is a radar site that can detect within the surrounding circle. The ○ (vehicles 1 and 2) are stealth UAVs capable of evading radar and are responsible for removing the radar sites. The △, ⋄, ▽ (vehicles 3, 4 and 5) do not have the stealth capability and cannot fly through radar zones. These UAVs are responsible for removing the targets marked with ×. These UAVs are also only capable of flying 60% of the maximum speed of the stealth UAVs.

The first plot (left) shows the original environment and initial allocation of UAVs to targets. Note the UAVs are assigned a sequence of tasks rather than just a single task, as would be done in a "short-sighted" approach to the allocation problem [3]. Note that the targets are divided amongst the UAV's to minimize mission time.

The second plot shows how the fleet is re-assigned when UAV 1 removes radar site R1. The trajectory for UAV 4 is modified to fly through the previously obstructed radar zone reducing the mission time for this UAV. As a result, UAV 4 trades target T2 to UAV 5 and takes over target T4 in order to reduce the total mission time for the fleet. This demonstrates both the complete cooperation among the fleet of UAVs and the ability of this method to make decisions regarding not just the current task, but future tasks that contribute to the total cost of the mission.

In the third plot, UAV 2 has removed radar site R2 which again allows UAV 4 to shorten its trajectory. However, a new radar site R5 is also detected by UAV 1, which results in a reallocation of tasks for the stealth UAVs. UAV 1 is now tasked with removing R5, while UAV 2 receives R3 in addition to the previous task of removing R4.

This simple simulation was developed to demonstrate the capability of the UAV allocation problem presented in this paper to adapt to changes in the environment. The sub-team re-allocation method used in the simulation allows fast reassignment of tasks within sub-teams which leads to cooperation between the UAV in completing the total mission. The next section investigates techniques for accounting for the uncertainty in the environment while developing the initial plan.

3 Optimization For A Stochastic Environment

This section presents three formulations of the allocation problem that are progressively more cognizant of its stochastic properties. The first is a purely deterministic formulation that assumes no UAVs are lost. The second is a deterministic equivalent that models the probability of UAV loss, without taking into consideration the reduction in this probability that comes from destroying anti-aircraft defenses [8, 9]. The third is a new stochastic optimization that models both the probability of UAV loss and the ability of cooperation to reduce the probability of loss.

These formulations extend the basic minimum completion time formulation given in Eq. 1. The waypoint permutations are expanded to include the UAV's terminal points. They also apply the constraints that force the following variables to take their desired values; V_{wvp} is 1 if waypoint w is visited by vehicle v on its p^{th} permutation and 0 if not, t_{w} is the time that waypoint w is visited, $t_{0\mathrm{v}}$ is the aircraft's time of departure from its starting point, and T_{dv} is the length of time after its departure that vehicle v visits its d^{th} waypoint. In

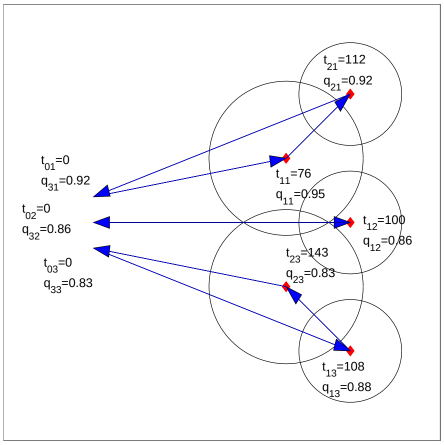

Fig. 3: Example of a purely deterministic allocation. t_{dv} gives the time at which point d is reached on UAV v's mission. q_{dv} is the probability that point d is reached on UAV v's mission.

order to emphasize distinctions between formulations, we assign variable names with tildes to probabilities and scores whose calculation neglects the coupling effects between UAV missions, and variable names without tildes to their equivalents whose calculation takes this coupling into consideration.

The results of applying these formulations to the same allocation problem are presented, and the level of anti-aircraft defense threat in the environment is varied to understand its effects. The expected score of each approach is calculated in Section 4. The less sophisticated approaches are shown to achieve worse expected scores for simple problems, and to be unable to plan successful missions for more difficult scenarios. The full stochastic formulation is shown to achieve the highest expected score.

3.1 Purely Deterministic Formulation

The first modified formulation extends the cost function of Eq. 1 to include a score $\tilde{s}_{\mathrm{dvp}}$ associated with each waypoint in order to balance completion time against the value of waypoints allocated to UAVs

$$\max_{x_{\mathrm{vp}},t_{0\mathrm{v}}} J_2 = \sum_{\mathrm{d}=1}^{n_{\max}} \sum_{\mathrm{v}=1}^{N_V} \sum_{\mathrm{p}=1}^{N_P} \tilde{s}_{\mathrm{dvp}} x_{\mathrm{vp}} - \alpha_1 \bar{t} - \frac{\alpha_2}{N_V} \sum_{\mathrm{v}=1}^{N_V} (t_{0\mathrm{v}} + T_{\mathrm{n_{max}v}}) \qquad (2)$$

where $\tilde{s}_{\mathrm{dvp}}$ an input to the allocation problem representing the score of the d^{th} destination of vehicle v on its p^{th} permutation, and the weights α_1 and α_2 are selected to weight completing the mission quickly against planning longer missions that visit more waypoints. The requirement that every point be visited is relaxed, and this formulation neglects the possibility of UAV attrition. This formulation tends to result in "optimistic" plans, in which risk is ignored in favor of high scores.

An example of a mission plan found with this purely deterministic formulation is shown in Fig. 3. The central waypoint has a score of 100 points, and the other waypoints have a score of 10. The UAVs each receive a score of 50 for returning to their starting point, representing the perceived value of the UAVs relative to the waypoints. The full results in Tables 1–3 are discussed in Section 4.

In this work, the probability that a UAV is destroyed is calculated as proportional to the length of its path within the anti-aircraft defense's range. In the nominal threat level case, the constant of proportionality was chosen so that a path to the center of the smaller anti-aircraft defense would have a probability of survival of 0.96. The formulations were also applied in environments in which the nominal constant of proportionality was multiplied by factors of 3 and 7, respectively. These particular selections are arbitrary, but the results of this comparison illustrate important trends in the performance as the threat level increases.

With nominal threat levels, this formulation gave a probability of 0.86 that the high value target at center would be reached by the UAV to which it was allocated. When the probability of destruction on each leg was increased by a factor of 3 and 7, the probability of reaching the high value target was 0.57 and 0.25 respectively (see Table 3). This shows that in well-defended environments, the deterministic formulation plans missions that are highly susceptible to failure.

3.2 Deterministic Equivalent of the Stochastic Formulation

Similar to [8, 9], this second form models the threat that each waypoint poses to UAVs as a fixed quantity, so that destroying it does not decrease the risk to other UAVs. This reduces the problem to multiplying the score associated with each waypoint along a UAV's mission by the probability that the UAV reaches that waypoint. This calculation can be done for every permutation before the optimization is performed, so no probabilities are explicitly represented in the optimization program itself. This approach allows sophisticated relationships between survival probability and radar exposure to be used. Voronoi diagrams can be used as a basis for path approximations in order to minimize radar exposure [10], and time and probability values for several different paths can be provided for each ordering of waypoints.

Let $\tilde{q}_{\mathrm{dvp}}$ be the probability that vehicle v reaches the d^{th} destination on its p^{th} permutation, and let $d = 0$ correspond to the vehicle's starting position. Then $\tilde{q}_{0\mathrm{vp}} = 1.0$ for all permutations, and

$$\tilde{q}_{\mathrm{dvp}} = \tilde{q}_{(\mathrm{d}-1)\mathrm{vp}} \prod_{\mathrm{w}=1}^{N_W} \tilde{q}_{\mathrm{dvwp}} \qquad (3)$$

where $\tilde{q}_{\rm dvwp}$ is the probability that an anti-aircraft defense at waypoint w does not shoot down UAV v between its $(d-1)^{\rm th}$ and $d^{\rm th}$ destinations. Then, the cost function of Eq. 2 can be modified to use the deterministic equivalent of the score $\tilde{q}_{\rm dvp}\tilde{s}_{\rm dvp}$

$$\max_{x_{\rm vp},t_{\rm 0v}} J_3 = \sum_{\rm d=1}^{n_{\max}} \sum_{\rm v=1}^{N_V} \sum_{\rm p=1}^{N_P} \tilde{q}_{\rm dvp}\tilde{s}_{\rm dvp} x_{\rm vp} - \alpha_1 \bar{t} - \frac{\alpha_2}{N_V} \sum_{\rm v=1}^{N_V} (t_{\rm 0v} + T_{\rm n_{max}v}) \quad (4)$$

where $\tilde{q}_{\rm dvp}\tilde{s}_{\rm dvp}$ is evaluated in the cost estimation step, and is passed into the optimization as a parameter. Fig. 4 shows allocation plans from the deterministic equivalent formulation on a simple example. This formulation includes a notion of risk, but does not recognize the ability of UAVs to cooperate to decrease the probability of attrition. As the threat level of the environment increases, this formulation tends to result in "pessimistic" plans, in which some of the waypoints are not visited. This occurs when the contribution to the expected score of visiting the remaining waypoints is offset by the decrease in expected score of doing so due to a lower probability of surviving to return. The ability to reduce risk through cooperation can be captured by evaluating the actual risk during optimization as a function of the waypoint visitation precedence.

3.3 Stochastic Formulation

This section presents a new stochastic optimization formulation that also maximizes the expected score. This optimization will be shown to exploit phasing by attacking the anti-aircraft defenses before the high value targets, and to preserve the survival of the UAV which visits the high value target. To determine whether an anti-aircraft defense is in operation while a UAV flies within its original range, the waypoint visitation precedence is evaluated. If the time that UAV v begins the leg leading to its $d^{\rm th}$ destination is less than the time waypoint w is visited, then waypoint w is considered to threaten the UAV on this leg from $d-1$ to d, and the binary decision variable $A_{\rm dvw}$ is set to 1 to encode this waypoint visitation precedence. The logical equivalence

$$A_{\rm dvw} = 1 \Leftrightarrow t_{\rm 0v} + T_{\rm (d-1)v} \le t_{\rm w} \quad (5)$$

can be enforced with the constraints

$$\begin{aligned} t_{\rm 0v} + T_{\rm (d-1)v} &\le t_{\rm w} + M(1 - A_{\rm dvw}) + \epsilon \\ t_{\rm w} &\le t_{\rm 0v} + T_{\rm (d-1)v} + M(1 - A_{\rm dvw}) + \epsilon \end{aligned}$$

where ϵ is a small positive number, M is a large positive number. With this precedence information available, constraints which evaluate the probability $q_{\rm dv}$ that vehicle v survives to visit the $d^{\rm th}$ waypoint on its mission can be formulated. The probability $\tilde{q}_{\rm dvw}$ of vehicle v not being destroyed on the leg leading to its $d^{\rm th}$ destination by an intact air defense at waypoint w for the selected permutation is evaluated as

$$\tilde{q}_{\rm dvw} = \tilde{q}_{\rm dvpw} x_{\rm vp} \quad (6)$$

If waypoint w is visited before the vehicle starts the leg to destination d, then the anti-aircraft defense at w is assumed not to threaten the vehicle. Thus the actual probability $q_{\rm dvw}$ that vehicle v is *not* destroyed by an anti-aircraft defense at w is 1. Otherwise, it is $\tilde{q}_{\rm dvw}$

$$q_{\rm dvw} \le \tilde{q}_{\rm dvw} + M(1 - A_{\rm dvw}) \ \text{ and } \ q_{\rm dvw} \le 1 \quad (7)$$

The actual probability $q_{\rm dv}$ of reaching each destination can be found by evaluating Eq. 3 in terms of the actual probability of surviving each anti-aircraft defense $q_{\rm dvw}$

$$q_{\rm dv} = q_{\rm (d-1)v} \prod_{\rm w=1}^{N_W} q_{\rm dvw} \quad (8)$$

where again, $d = 0$ corresponds to the vehicle's starting position and $q_{\rm 0v} = \tilde{q}_{\rm 0v} = 1.0$. Because Eq. 8 is nonlinear in decision variables $q_{\rm dvw}$ and $q_{\rm dv}$, it cannot be included directly in the formulation, but can be tranformed using logarithms as

$$\log q_{\rm dv} = \log q_{\rm (d-1)v} + \sum_{\rm w=1}^{N_W} \log q_{\rm dvw} \quad (9)$$

While this form accumulates the effects of each of the anti-aircraft defense sites on the survival probability over each leg of the mission, it only provides $\log q_{\rm dv}$. Evaluating the expected score requires $q_{\rm dv}$, and this can be recovered approximately as $q'_{\rm dv}$ by raising 10 to the exponent $\log q_{\rm dv}$ using a piecewise linear function which can be included into a MILP accurately using 3 binary variables, since the exact function is nearly linear in the range of interest where probabilities are above 0.3 [11].

The expectation of the mission score is then found by summing waypoint scores multiplied by the probability of reaching that waypoint. If the score of the $d^{\rm th}$ waypoint visited by vehicle v in its $p^{\rm th}$ permutation is $\tilde{s}_{\rm dvp}$, then the expectation of the score $s_{\rm dv}$ that will be received from visiting the waypoint is

$$\forall p \in \{1, 2, \ldots, N_P\} \ : \ s_{\rm dv} \le q'_{\rm dv}\tilde{s}_{\rm dvp} + M(1 - x_{\rm vp}) \quad (10)$$

and the objective of the stochastic formulation is

$$\max_{x_{\rm vp},t_{\rm 0v}} J_4 = \sum_{\rm d=1}^{n_{\max}} \sum_{\rm v=1}^{N_V} s_{\rm dv} - \alpha_1 \bar{t} - \frac{\alpha_2}{N_V} \sum_{\rm v=1}^{N_V} (t_{\rm 0v} + T_{\rm n_{max}v}) \quad (11)$$

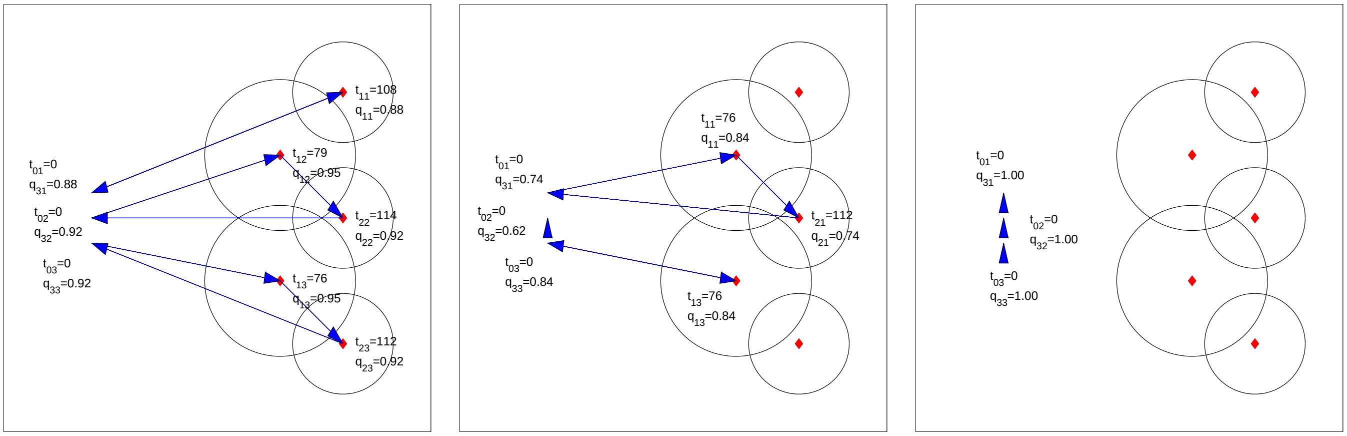

Fig. 4: Example Deterministic Equivalent Allocations. Nominal probabilities of destruction on the left, increased by factor of 3 in the middle, and increased by factor of 7 on the right.

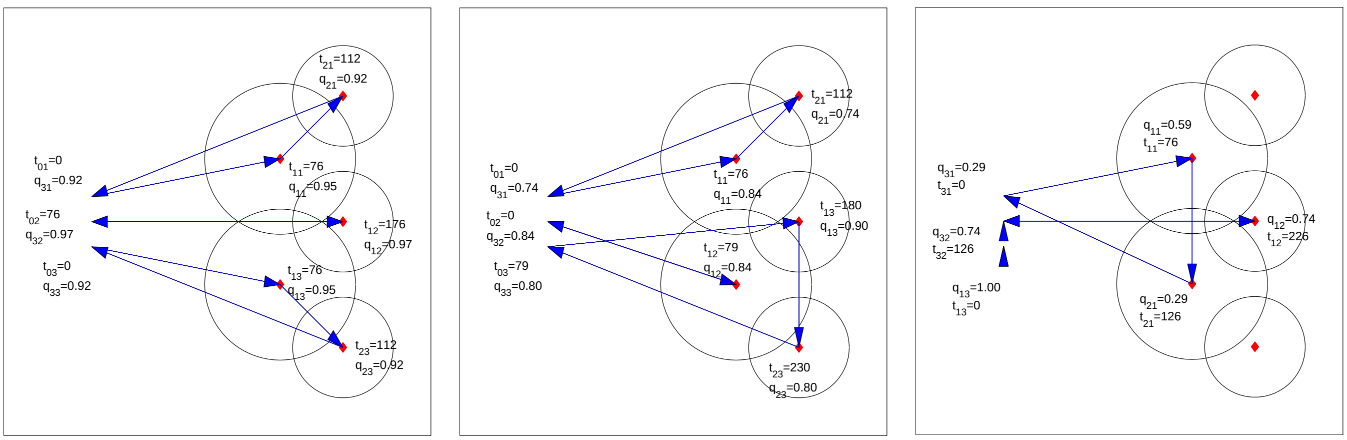

Fig. 5: Example maximum expected score allocation. Nominal probabilities of destruction on the left, increased by factor of 3 in the middle, and increased by factor of 7 on the right.

Optimal allocations for this problem are in Fig. 5, and a careful analysis shows that it recovers phasing (*e.g.,* $t_{02} = 76$) and preserves the UAV that visits the high value target. As the threat level in the environment increases, the upper and lower waypoints are ignored.

4 Results

4.1 Nominal Environment

After the coordination problem was solved for nominal threat values using the three formulations described above, the resulting allocation solutions were evaluated using the model of the stochastic formulation of Section 3.3. The resulting expected score, mission completion time, and probability of survival of the three formulations is compared in Table 1. The computation time of each formulation is also shown. Note that the expected score of the purely deterministic and stochastic formulations is very different, although the waypoint combinations assigned to each UAV are the same and the allocation differs mainly in timing. This emphasizes the importance of timing of activities.

While some improvement over the completely deterministic formulation is seen in the deterministic equivalent formulation, the stochastic formulation achieves the highest expected score. This formulation also does the best job of protecting the survival of the UAV that visits the high value target. It is, however, the most computationally demanding formulation.

4.2 High Threat Environments

The results of applying all three formulations in high threat environments are shown in Tables 2 and 3, which indicate that (in high threat environments) the completely deterministic and deterministic equivalent approaches are incapable of recovering a higher expected score than would be achieved by keeping the UAVs at their base. Also, these two formulations are not capable of designing a plan that is likely to reach the high value target.

4.3 Results on Larger Problem

It is typical of these problems that the expected score formulation quickly finds a good answer that was close to optimal, then made very small improvements in the expected score for the rest of its solution time. To examine this in more detail, the expected score formulation was also applied to a large problem with 4 vehicles

Table 1: Results of several formulations in probabilistic environment (Nominal threat levels)

Formulation	Exp. Score	$\bar{t}$	Computation Time (s)
Min. Time	251.3	219.5	6.5
Deterministic Equiv.	263.8	219.5	7.0
Stochastic	273.1	276.5	27.1

Table 2: Expected scores in threatening environments. Various probabilities of destruction (nominal, and 3 & 7 times higher).

	Expected Score		
Formulation	Nominal	×3	×7
Min. Time	251.3	173.1	81.4
Deterministic Equiv.	263.7	219.6	150.0
Stochastic	273.15	239.9	208.7

Table 3: Probability of reaching high value target. Various probabilities of destruction (nominal, and 3 & 7 times higher).

	Probability		
Formulation	Nominal	3	7
Min. Completion Time	0.86	0.57	0.25
Deterministic Equiv.	0.92	0.74	0.00
Stochastic	0.97	0.9	0.74

and 11 targets, and the expected score of the incumbent solution was recorded over time during the optimization process. This optimization was not solved to completion, but achieved a maximum score of about 342 in 60 minutes. However, an incumbent solution with an expected score of about 331 was found in only 18 seconds. In this problem, each vehicle can visit 2 waypoints.

The results are shown in Fig. 6. Note that UAV 2 delays its departure just long enough that 5 of the anti-aircraft defenses have been destroyed. UAV 2 then visits waypoint A at the same time ($t = 108$) as UAV 1 visits waypoint B. Also note that waypoints A and B have been selected as the two that are farthest apart, so that UAV 2 can reach the A without being significantly threatened by B. This preserves UAV 2's survival and minimizes completion time.

5 Conclusions

The paper presents a new formulation of the stochastic weapon-task assignment problem. This formulation is shown to be an extension of previous work because it accounts for the coupling between each UAV's failure probability and the missions assigned to all other UAVs. To maximize the expected mission score, it is shown that this stochastic formulation results in coordination plans that optimally exploit the coupling effects of co-operation between the UAVs to improve survival probabilities. The allocation optimization recovers real-world planning strategies, such as phasing of the UAVs, and yields significant improvements over approaches that do not correctly account for UAV attrition.

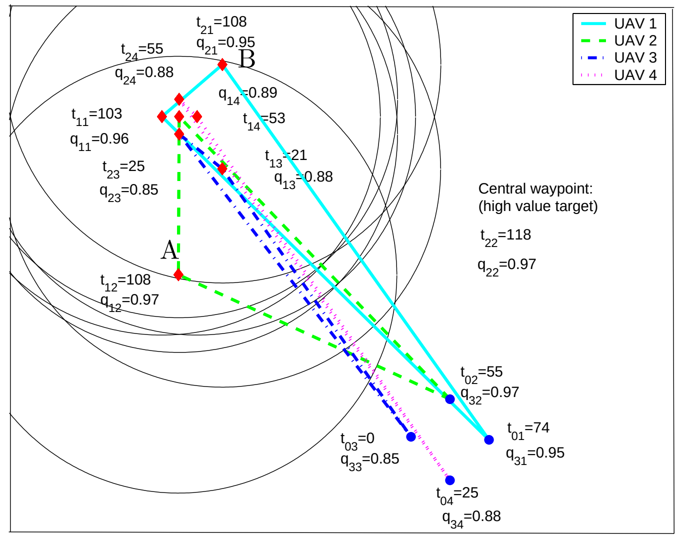

Fig. 6: Example Large Allocation Problem. Vehicle starting positions shown with •. Seven of the waypoints represent anti-aircraft defenses, while the 8$^{\text{th}}$, at the center of the tight cluster of waypoints, is a high value target that presents no threat.

References

[1] P. R. Chandler, S. Rasmussen, "UAV Cooperative Path-Planning," Proc. of the *AIAA GNC,* Aug. 14–17, 2000. AIAA-2000-4370.

[2] J. Wohletz, "Cooperative, Dynamic Mission Control for Uncertain, Multi-Vehicle Autonomous Systems" special session presented at the IEEE *CDC*, Dec. 2002.

[3] P. Chandler, M. Pachter, "Hierarchical Control for Autonomous Teams" Proc. of the *AIAA GNC,* Aug., 2001.

[4] J. S. Bellingham, M. J. Tillerson, A. G. Richards, J. P. How, "Multi-Task Assignment and Path Planning for Cooperating UAVs," *Conference on Cooperative Control and Optimization,* Nov. 2001.

[5] A. G. Richards, J. P. How, "Aircraft Trajectory Planning with Collision Avoidance using Mixed Integer Linear Programming," *ACC,* May 2002.

[6] T. Schouwenaars, B. DeMoor, E. Feron and J. How, "Mixed integer programming for safe multi-vehicle cooperative path planning," *ECC*, Sept. 2001.

[7] J. S. Bellingham, A. G. Richards and J. P. How, "Receding Horizon Control of Autonomous Aerial Vehicles", *ACC,* Anchorage, AK, May 2002.

[8] R. A. Murphey, "An approximate algorithm for a weapon target assignment stochastic program," In *Approximation and Complexity in Numerical Optimization: Continuous and Discrete Problems.* Kluwer Acad. Publ., 1999.

[9] J. L. Ryan, T. G. Bailey, and J. T. Moore, "Reactive tabu search in unmanned aerial reconnaissance simulations," In D. J. Medeiros et al., editor, *Proceedings of the 1998 Winter Simulation Conference*, 1998.

[10] T. McLain, P. Chandler, S. Rasmussen, and M. Pachter. "Cooperative control of UAV rendezvous," In *Proceedings of the American Control Conference*, pages 2309–2314, Arlington, VA, June 2001.

[11] J. S. Bellingham, *Coordination and Control of UAV Fleets using Mixed-Integer Linear Programming,* SM Thesis, MIT Department of Aeronautics and Astronautics, Aug. 2002.