Advanced Guidance Algorithms for Spacecraft Formation-keeping*

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Abstract

This paper presents advanced formation-keeping guidance algorithms that use linear programming (LP) to determine fuel-optimal control inputs and state trajectories. The overall formation-keeping problem is analyzed in terms of two key issues: (i) what dynamics model should be used to specify the desired state to maintain a passive aperture; and (ii) what dynamics model should be used in the LP to represent the motion about this state. Several linearized models of the relative dynamics are considered in this analysis, including Hill's equations for circular orbits, modified linear dynamics that partially account for the J_2 effects, and Lawden's equations for eccentric orbits. A controller is developed for formation-keeping using each of these models. A modified LP formulation is presented to include robustness to sensor noise while ensuring a feasible solution. The guidance algorithms are implemented in numerous very detailed nonlinear simulations that demonstrate effective control in the presence of all expected disturbances and sensor noises. The average fuel cost for the formation-keeping maneuvers over a two week simulation is on the order of 4 mm/s per orbit.

1 Introduction

A large number of future planned space missions are based on a new approach that will use coordinated microsatellites to provide flexible, low-cost access to space [1, 2]. However, to achieve these future mission goals, several guidance, navigation, and control challenges must first be addressed. For example, very tight coordination, control, and monitoring of the distributed vehicles in the cluster will be required to achieve the stringent payload pointing requirements for a synthetic aperture radar mission, such as TechSat 21 [3]. Much of the research for cluster dynamic modeling and control has focused on the design of *passive apertures*, which are (short baseline) periodic formation configurations that provide good, distributed, Earth imaging while reducing the tendency of the vehicles to drift apart. These passive apertures can be designed using the closed-form solutions provided by Hill's equations [4, 5] (also known as the Clohessy-Wiltshire equations), which assume a circular reference orbit. There has been further analysis to develop apertures that are insensitive to differential J_2 disturbances [6, 7] and reference orbit eccentricity [8].

The purpose of this paper is to present extensions of a recently proposed approach to formation flying control that uses linear programming (LP) to solve for both the fuel-optimal control inputs and trajectories. In particular, the LP approach is generalized to include various linearized models of the relative spacecraft dynamics. The model set includes the basic Hill's equations for circular orbits, a modified hybrid set of dynamics that partially account for the J_2 disturbances, and Lawden's equations for eccentric orbits. Extensive nonlinear simulations are performed to compare the effectiveness of the control algorithms based on these different dynamics. Both circular and eccentric orbits are used in these simulations. Based on these results, this paper addresses the important questions of which dynamics model should be used to define the desired state to maintain a passive aperture and which model should be used in the LP algorithm. The LP algorithm is also modified to add robustness to sensor noise while ensuring an always feasible solution. The complete guidance algorithm is implemented in a detailed two-week nonlinear simulation that demonstrates effective control in the presence of all expected disturbance forces with a fuel cost of 4 mm/s per orbit. The reduction in fuel cost over previous results is achieved through various improvements, such as reduction in the level of the sensor noise, improved dynamic models, and better algorithm formulation and implementation.

2 Relative Dynamics

The linear programming control technique requires linearized relative dynamics between a satellite and some reference orbit. The reference orbit can be fixed on another satellite in the formation, the formation center, or a virtual satellite. Three different types of dynamics are investigated for the LP control approach. The most commonly used linearized dynamics for relative control

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are Hill's dynamics, which assume a circular reference orbit and a central gravitational force [9]. The coordinate frame for Hill's is specified as (x) radial away from the earth, (y) in-track, perpendicular to x and in the direction of velocity, and (z) crosstrack, which completes the right-hand system.

The second form of dynamics is very similar to Hill's, but has been modified to include linearized effects of the J_2 gravitational perturbations. The dynamics presented here are actually a combination of the work of Refs. [7, 10]. The linearized dynamics including J_2 effects are

$$\begin{aligned}\ddot{x} &= 2nc\dot{y} + (5c^2-2)n^2x + f_x \\ \ddot{y} &= -2nc\dot{x} + f_y \\ \ddot{z} &= -(3c^2-2)n^2z + f_z + 2Anca_{\text{ref}}\cos\alpha\sin\theta_{\text{ref}}\end{aligned} \quad (1)$$

$$\begin{aligned}\text{with}\quad s &= \frac{3}{8}J_2\left(\frac{R_{\text{earth}}}{a_{\text{ref}}}\right)^2(1+3\cos(2i_{\text{ref}})) \\ c &= \sqrt{s+1} \\ A &= \frac{3}{2}J_2 n\left(\frac{R_{\text{earth}}}{a_{\text{ref}}}\right)^2\sin^2(i_{\text{ref}})c_2 \\ c_2 &= \frac{\rho}{a_{\text{ref}}}\end{aligned} \quad (2)$$

and n is the mean motion, a_{ref} is the semi-major axis, i_{ref} is the inclination, and θ_{ref} is the true anomaly of the reference orbit. α is the formation phasing angle and ρ is the formation radius [10]. The f's correspond to control inputs and any other disturbance forces. The crosstrack J_2 disturbance is modeled as an input disturbance, w, in the LP formulation as shown in [11].

Note that if we set $J_2 = 0$, then s and c also equal zero and these dynamics simplify to Hill's equations. The reason this set of dynamics is composed from two separate sources is that the in-plane dynamics in Ref. [10] require an iteration on a parameter to speed up the orbital motion in the dynamics where as Ref. [7] provides a direct calculation for the parameter c to achieve the same effect. Conversely, the out-of-plane J_2 disturbance in Ref. [7] requires several calculations involving both relative and absolute measurements to determine the disturbance, whereas the model in Ref. [10] only requires a relatively straightforward calculation. This combination appears to give the best fit to the nonlinear orbital simulations.

The third form of the dynamics, for eccentric reference orbits, was first developed by Lawden [12]. From [13], the radius and angular velocity of the formation center are written as

$$|\vec{R}_{\text{fc}}| = \frac{a_{\text{ref}}(1-e^2)}{1+e\cos\theta} \text{ and } \dot{\theta} = \frac{n(1+e\cos\theta)^2}{(1-e^2)^{3/2}} \quad (3)$$

With these expressions, the dynamics for elliptical orbits can be written in the time domain, but writing the equations as a function of the true anomaly, θ, provides a more natural description. This is because both the radius of the orbit and angular velocity are functions of the true anomaly. The transition from time domain to θ-domain requires the following changes in derivatives $\dot{(\cdot)} = (\cdot)'\dot{\theta}$; $\ddot{(\cdot)} = (\cdot)''\dot{\theta}^2 + \dot{\theta}\dot{\theta}'(\cdot)'$ [8]. Lawden's equations of motion are then

$$\frac{d}{d\theta}\begin{bmatrix} x' \\ x \\ y' \\ y \end{bmatrix} = \begin{bmatrix} \frac{2e\sin\theta}{1+e\cos\theta} & \frac{3+e\cos\theta}{1+e\cos\theta} & 2 & \frac{-2e\sin(\theta)}{1+e\cos\theta} \\ 1 & 0 & 0 & 0 \\ -2 & \frac{2e\sin\theta}{1+e\cos\theta} & \frac{2e\sin\theta}{1+e\cos\theta} & \frac{e\cos\theta}{1+e\cos\theta} \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x' \\ x \\ y' \\ y \end{bmatrix} + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} f_x \\ f_y \end{bmatrix} \quad (4)$$

$$\frac{d}{d\theta}\begin{bmatrix} z' \\ z \end{bmatrix} = \begin{bmatrix} \frac{2e\sin\theta}{1+e\cos\theta} & \frac{-1}{1+e\cos\theta} \\ 1 & 0 \end{bmatrix}\begin{bmatrix} z' \\ z \end{bmatrix} + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} f_z \end{bmatrix} \quad (5)$$

Again, the in-plane dynamics are decoupled from the out-of-plane dynamics. The f's represent control as well as disturbance input forces. Note that the dynamics are a function of the true anomaly θ, so the system is linear time-varying.

3 Formation-keeping Control

Disturbances such as differential drag, J_2, and errors in the linearized dynamics will cause the satellite to drift from the designed periodic motion of passive apertures. As a result, a control scheme is required to maintain a state that results in the periodic motion. Linear programming can be used to develop fuel-optimal control inputs to move the satellite from the disturbed state back to the desired state or to maintain the satellite within some tolerance of the desired state. The tolerance is specified by an error box fixed to the desired state for the satellite. Based on current performance specifications, the error box size is 10% of the baseline distance between the satellites [3].

The formation-keeping problem is comprised of two issues. The first issue is what relative dynamics and initialization procedure should be used to specify the desired state to maintain the passive aperture formation. The desired state is shown in Fig. 1 as a diamond and the reference orbit position as a circle. The periodic motion followed in the absence of disturbances is also shown. The desired state is determined from the closed form solutions of the linearized dynamics and the initial conditions. Note that the closed-form solutions and initial conditions for periodic motion are slightly different (see

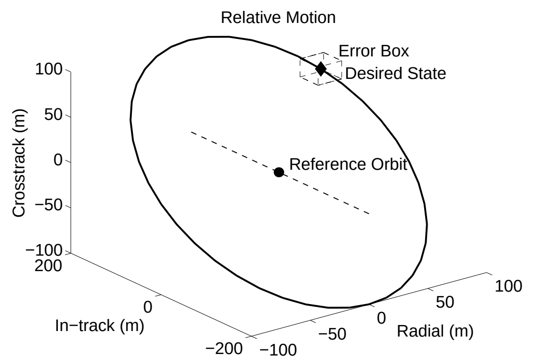

Fig. 1: Satellite motion relative to a reference orbit. Current position of the reference orbit is denoted with a circle, and the current desired relative position is with a diamond. An error box is centered on the diamond.

Refs. [7, 8, 9] for details) for each type of relative dynamics discussed in Section 2. The conditions for periodic motion for each set of dynamics are

$$\text{Hill's: } \frac{\dot{y}(0)}{x(0)} = -2n \tag{6}$$

$$\text{Lawden's: } \frac{\dot{y}(0)}{x(0)} = -\frac{n(2+e)}{(1+e)^{1/2}(1-e)^{3/2}} \tag{7}$$

$$\text{J}_2\text{: } \frac{\dot{y}(0)}{x(0)} = -2n\sqrt{1+s} \tag{8}$$

These initial conditions are then used in the corresponding closed-form solutions to determine the desired state at any other time. Note that each model accounts for different aspects of the fleet reference orbital motion (eccentricity) and disturbances (J_2). The desired state for each spacecraft is specified by a central coordinator, but the formation-keeping problem is distributed among the individual spacecraft. The simulations presented in Section 4 compare the impact of using each of these models to predict the desired state for various types of orbits.

The second issue for formation-keeping is which relative dynamics to use in the actual LP problem. The error box is fixed to the desired state as in Fig. 1. The desired state is centered in the error box, but the true state of the satellite will be disturbed from the desired state by differential drag, J_2, or other disturbances. The error state is then the difference between the current state and desired state relative to the reference orbit. The dynamics used in the LP are the dynamics relative to the desired state. Ref. [14] implements the time changing dynamics for eccentric orbits in the LP but shows that Hill's can be used with little fuel cost increase for small eccentricities ($e \leq 0.01$). Using the time-varying dynamics does not increase the size of the LP problem but formulating the problem is computationally more complex. Further investigation into the effect of using each form of dynamics in the LP is in Section 4.

3.1 LP problem formulation

The linear programming trajectory planning approach was presented in Ref. [11] to design fuel-optimized trajectories. Each of the dynamics presented in the Section 2 can be discretized and manipulated into the following form

$$y(k) = A(k)U_k + b(k) \tag{9}$$

The formation-keeping LP problem is to maintain a desired state to within some tolerance over n steps in time (or true anomaly), while minimizing a weighted sum ($c_j \geq 0$) of the $\|\cdot\|_1$ norm of the control inputs by each spacecraft. The control inputs are normalized, where 1 means the thruster is on for the entire time step and anything less is some fraction of the time step. To write this problem as a linear program, two slack variables are introduced that define the positive and negative parts of the control input

$$U_n = U_n^+ - U_n^-, \quad U_n^+ \geq 0,\ U_n^- \geq 0 \tag{10}$$

Using c_{ij} as the weight for the input from the j^{th} actuator at the i^{th} time step, define

$$C^T = [c_{00}\ c_{01}\ \ldots\ c_{nm}\ c_{00}\ c_{01}\ \ldots\ c_{nm}] \tag{11}$$

as the weights on each of the positive and negative parts of the control inputs. The formation-keeping problem can then be written as the linear program

$$J^* = \min_{U_n} C^T \begin{bmatrix} U_n^+ \\ U_n^- \end{bmatrix}$$

subject to

$$\begin{bmatrix} A(k) & -A(k) \\ -A(k) & A(k) \end{bmatrix} \begin{bmatrix} U_n^+ \\ U_n^- \end{bmatrix} \leq \begin{bmatrix} y_{\text{des}}(k) - b(k) + y_{\text{tol}} \\ -y_{\text{des}}(k) + b(k) + y_{\text{tol}} \end{bmatrix} \tag{12}$$

where y_{tol} specifies the size of the error box. At each time step k in the plan where the tolerance constraint is applied, two sets of constraints are included in the problem to constrain the satellite position to remain inside the error box.

3.2 Algorithm Initiation

As discussed previously, the first step in applying the LP technique in a spacecraft control system is to determine the desired state. An error box is fixed to the desired state to provide a position tolerance for the satellite. Fig. 2 shows an in-plane view of the error box. The deviation of the current position from the desired position (called the *error state*) is used to initiate the LP algorithm and determine the control inputs and trajectories to maintain the position tolerance throughout the plan horizon. At each time step in the controller, the error position state is calculated and used to determine (i) if control action is needed when a plan does not exist, (ii) if the control should continue to use the existing plan, or (iii) make a new plan. The method for determining each of these actions is discussed in the following.

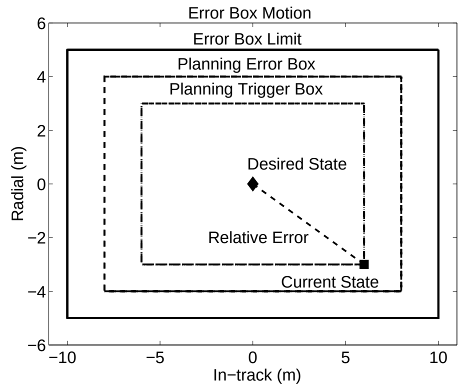

Fig. 2: In-plane view of error box. Three limits: plan trigger limit, planning error box constraint limit, hard performance (error box limit) constraint. The desired state is represented by the diamond while the current state is the square.

There are three parts to the error box. The error box *limit* is the largest box and represents the position tolerance not to be exceeded. The *planning* error box is slightly smaller and is the limit used in the constraints of the LP. The planning tolerance is slightly less because the dynamics used in the LP do not exactly match the nonlinear orbital dynamics and as a result the path followed by the satellite will not exactly match the designed trajectory. The smaller box allows some deviation in the path without exceeding the ultimate tolerances. The *planning trigger* box is the smallest box. When the state exceeds the trigger box and no plan exists, then a new plan is developed. When a plan does exists, the first half of the plan is implemented regardless of the current error position and then, if the position exceeds the planning error box, a new plan is formed. This limits the deviation from the designed trajectories. Of course, the relative sizes of the three boxes is a variable in the control scheme that can be used to increase or decrease performance at the expense or relief of fuel cost. The geometry of the error "box" is also a variable in the control implementation. The form of the dynamics suggests that using an oblate sphere rather than a cube could yield some performance benefits [15]. The sphere can be approximated in the LP using a polygon with a constraint for each side. Using a "sphere" would also avoid initial conditions to LP problems that result in higher fuel costs, such as when a satellite is near the corner of the error box with little room to maneuver. Future research will investigate this issue in more detail.

3.3 Sensor Noise

Any planned trajectory will rely heavily on the knowledge of the satellite's initial conditions, but the initial relative positions and velocities must be measured and will be noisy. Investigation of the impact of sensor noise on the LP control technique and a method for making formation-keeping plans robust to sensor noise is presented in Refs. [11, 16]. By considering several, m_{ic}, different initial conditions, the solution to the LP is made more robust to measurement errors by planning for the "worst case" response. This is achieved by minimizing the right hand side of Eq. 13 over the set $\mathcal{M} = \{1, \ldots, m_{ic}\}$ of possible initial conditions [16]. However, planning for the worst case can lead to feasibility problems. One solution to the feasibility problem is to reduce the plan horizon until the solution to the LP is feasible. The plan horizon could be reduced iteratively, but this would require attempting to solve multiple LP problems until a solution is achieved.

An alternative approach is to directly include a scaling of the error box size as a variable in the LP problem. The error box scaling variable, B_s, is heavily weighted in the cost function to prevent increasing the error box to achieve a solution with zero control inputs. Thus B_s would only be increased to scale the error box to achieve a feasible solution. The problem formulation is as follows

$$\begin{bmatrix} A(k) & -A(k) & -y_{\text{tol}} \\ -A(k) & A(k) & -y_{\text{tol}} \end{bmatrix} \begin{bmatrix} U_n^+ \\ U_n^- \\ B_s \end{bmatrix} \leq \min_{i \in \mathcal{M}} \begin{bmatrix} y_{\text{des}}(k) - b_i(k) \\ -y_{\text{des}}(k) + b_i(k) \end{bmatrix} \quad (13)$$

The scaling variable is constrained to be greater than one to prevent reducing the position tolerance below the original size in an attempt to minimize the heavily weighted scaling variable at the expense of increased control input in the cost.

4 Simulations

Several nonlinear simulations were performed using the FreeFlyer™ orbit simulator [17] in order to compare the effectiveness of the LP control method based on different forms of the dynamics. FreeFlyer™ is used to propagate the absolute states of both satellites. This simulator allows the option of including or excluding disturbances such as drag, lift, solar radiation pressure, and J_2. FreeFlyer™ easily interfaces with MATLAB™ where the control calculations are performed.

The simulations involve two similar satellites. One satellite acts as the formation center and serves as the reference orbit and the other satellite is initialized on a passive aperture. The reference orbit has a semi-major axis of 6900 km and inclination 35°. Simulations were performed for eccentricity $e \approx 0$ and $e = 0.005$. The passive aperture formed projects a 400×200 m ellipse on the orbital plane and oscillates with an amplitude of 100 m in the crosstrack direction, achieved through an inclination difference between the two satellites. This aperture is maintained through formation-keeping over two days. Each satellite is modeled as an Orion spacecraft based on current specifications for the Orion-Emerald mission [2].

Each satellite has a mass of 45 kg, but slightly different ballistic coefficients, resulting in a differential drag disturbance. The differential drag is modeled as a constant 5×10^{-8} m/s^2 disturbance acceleration in the LP. The satellite thrusters are restricted to provide a maximum acceleration of 0.003 m/s^2. The maximum thrust corresponds to turning on the thruster for the full time step. Sensor noise was also included as a true state plus white noise component. The noise is restricted to values less than 2 cm on position and 0.5 mm/s on velocity. These values are consistent with currently predicted noise levels using carrier phase differential GPS as the relative navigation sensor [18]. All disturbances (J_2, drag, solar radiation pressure, *etc.*) were included in all simulations, but the dynamics used in the controller are varied for comparison. The relative dynamics for the satellites are discretized on a 10.8 seconds time step. The LP plan horizon is half an orbit (approximately 45 minutes). Control inputs are allowed and state constraints applied every 108 seconds in the LP design. This reduces the LP size and decreases the solution time to 1–5 seconds.

Due to the stochastic nature of the simulations resulting from the sensor noise, each specific simulation in the following discussion was run three times, for a total of 24, two day simulations. There are two main parts that were varied for the simulations. The first part is the dynamics and resulting closed form-solutions and initialization, labeled in the Table as **Rel Dyn**. Only Lawden's and the J_2 dynamics are varied in this part because Hill's does not provide a fuel efficient desired state in the presence of J_2. In fact, using Hill's results in a fuel cost of approximately 300 mm/s per orbit in the presence of J_2. The second part varied is the dynamics used in the LP, labeled **LP Dyn**. All three forms of dynamics are used in the LP.

Table 1 summarizes the average fuel cost for formation-keeping using the various forms of the dynamics. The simulation results show that for nearly circular orbits, the J_2 dynamics provide the most fuel efficient results, with the other combinations of dynamics resulting in only a minimal increase in fuel cost. The correction of the mean motion with the parameter c or inclusion of eccentricity in both the relative dynamics and periodicity conditions in Eq. 8 lead to similar results and similar fuel cost savings. The improvement in using J_2 dynamics comes mostly from the inclusion of the secular crosstrack disturbance which is unmodeled in the other dynamics.

However, for a slightly eccentric reference orbit, the J_2 dynamics no longer provide an accurate description of the dynamics for determining the desired state. This degradation is a result of the fact that the J_2 dynamics still assume a circular reference orbit. Refs. [8, 14] have shown the significance of ignoring eccentricity. These simulations also confirm the results in Ref. [14] that using Hill's in the LP formulation does not significantly increase fuel cost. For the eccentric orbit, using Lawden's equations to specify the desired state and using the J_2 dynamics for the LP provides approximately 50% fuel cost reduction. This combination captures the orbit eccentricity in the prediction of the desired state and knowledge of the J_2 disturbance in the LP. A single simulation for each case discussed above was performed over a two week period to verify the control effectiveness over long time periods. The fuel cost numbers are within the uncertainty bounds of those presented in Table. 1. An example simulation over two weeks with $e = 0.005$ and using Lawden's equations to specify the desired state and the J_2 dynamics in the LP is shown in Figs. 3 and 4. Fig. 3 shows the relative motion between the two satellites while Fig. 4 shows the error box motion for a one day period during the simulation in order to observe the motion inside the error box. The average fuel cost for this simulations was 4.0 mm/s per orbit.

Table 1: Formation-keeping fuel comparison using each set of dynamics.

Rel Dyn	LP Dyn	$e \approx 0$ mm/s/orbit	$e = 0.005$ mm/s/orbit
J_2	J_2	5.7 ± 0.5	8 ± 2
Lawden	Hill's	7.7 ± 0.5	5 ± 1
Lawden	Lawden	7.6 ± 0.5	5 ± 1
Lawden	J_2	6.1 ± 0.5	4.0 ± 0.5

The fuel cost for formation-keeping has been significantly reduced from previous work in Refs. [11, 14, 16] through advancements in the guidance algorithm and reduction in sensor noise. Specific improvements are: 1) selection of the best dynamics to determine the desired state to maintain a passive aperture, 2) inclusion of the linearized J_2 effects in LP dynamics model, and 3) relaxation of the position tolerance variable to allow always feasible solutions. The inclusion of J_2 effects in the dynamics increases the accuracy of the model for developing trajectories. This means less replanning due to deviations from the designed trajectory, which also reduces fuel cost. The reduction in the sensor noise level and reformulating the problem for an always feasible solution allow longer plan horizons, which also reduces fuel cost. Further examination of the error box shape and size could lead to even further fuel savings.

5 Conclusions

This paper presents an advanced formation guidance and control strategy based on linear programming to determine fuel-optimal trajectories for formation-keeping. The overall formation-keeping problem is shown to consist of two key issues: (i) the selection of the dynamics

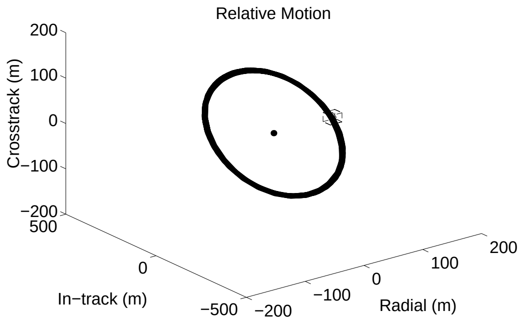

Fig. 3: Relative motion of satellite during the two week simulation. The circle represents the reference orbit and the diamond enclosed the box represents an example of the desired state and error box.

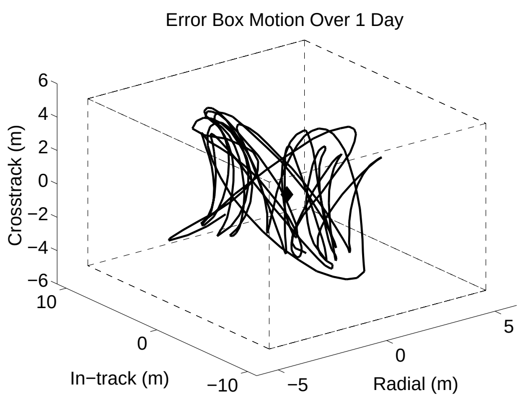

Fig. 4: Error box motion of satellite during simulation over a one day period. The diamond in the center of the box represents the desired state for maintaining the aperture.

to specify the desired state to maintain a passive aperture; and (ii) what dynamics should be used in the LP to represent the motion about this state. Several linearized dynamic models are compared in an extensive set of nonlinear simulations. The simulations indicate that the selection of the dynamics model is critical for determining the desired state, however the fuel cost is less sensitive to the dynamics model used in the LP. For nearly circular orbits, the linearized J_2 dynamics provide the lowest fuel cost. However, for non-circular orbits with $e \approx 0.005$, the J_2 dynamics do not provide a sufficiently accurate plant model and the most effective control uses Lawden's equations to specify the desired state and the J_2 dynamics in the LP controller. The simulations show that, with this combination, the LP guidance method provides an effective formation-keeping control strategy in the presence of all disturbances and sensor noise (estimated fuel cost of 3-5 mm/s per orbit). The paper also discusses modifications to the LP formulation to increase robustness to sensor noise while providing always feasible solutions. The result is a very flexible and effective optimization framework that addresses many of the control issues associated with formation flying spacecraft.

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